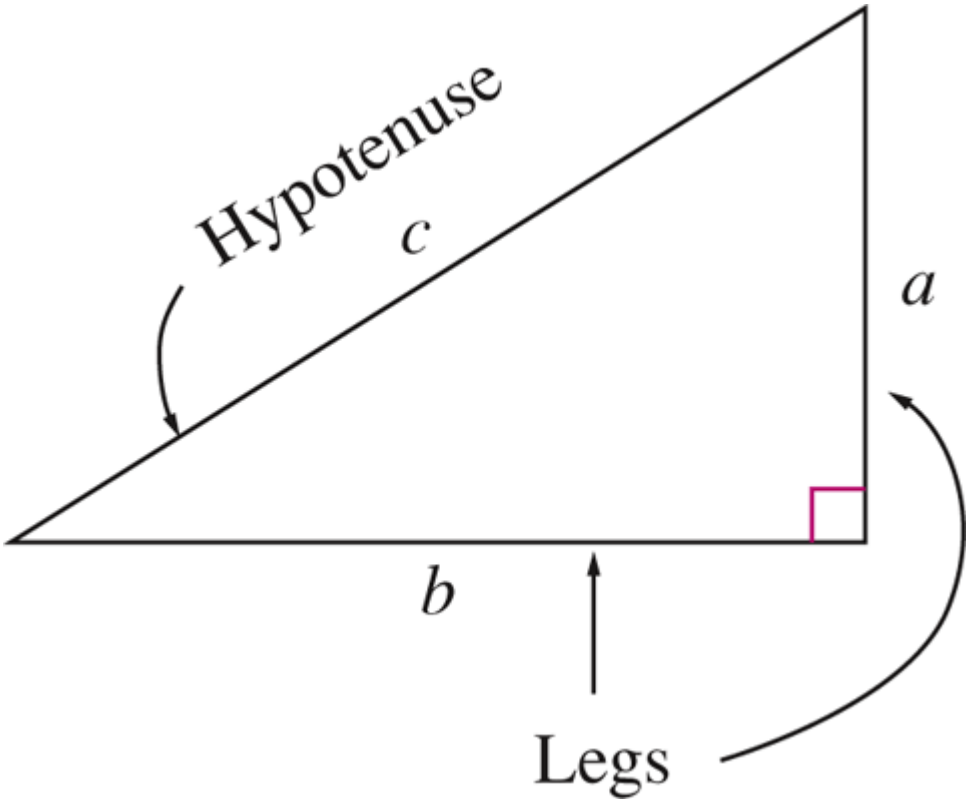
Section 13-3 The Pythagorean Theorem, Distance Formula, and Equation of a Circle

Students will be able to understand and explain

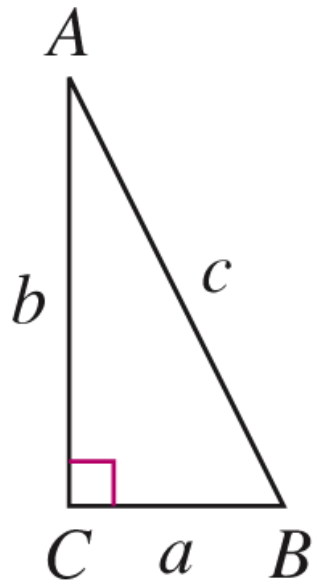
- Pythagorean theorem and its converse.
- Distance formula.
- Equation of a circle.

Parts of a Right Triangle



Pythagorean Theorem

Given a right triangle with legs *a* and *b* and hypotenuse *c*, $c^2 = a^2 + b^2$.



If $\overline{BC} = 3$ cm and $\overline{AC} = 4$ cm, what is the length of $\overline{AB}$?

$3^2 + 4^2 = c^2$

$9 + 16 = c^2$

$25 = c^2$

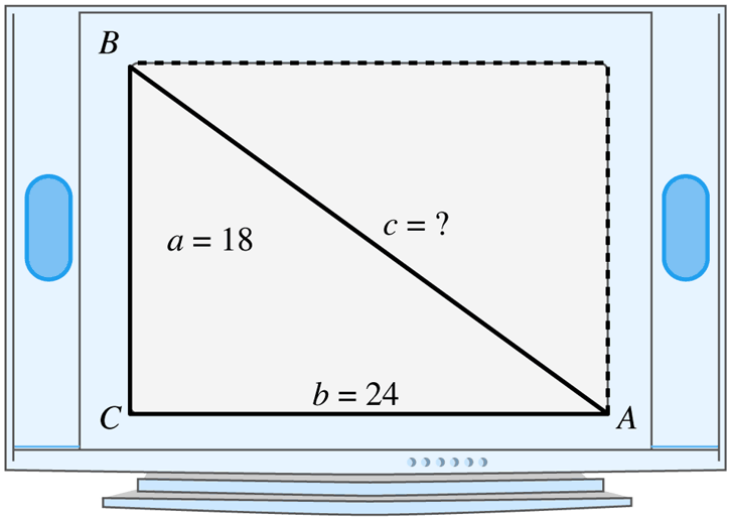
$5 = c$

Converse of the Pythagorean Theorem

If ΔABC is a triangle with sides of lengths *a*, *b*, and *c* such that $c^2 = a^2 + b^2$, then ΔABC is a right triangle with the right angle opposite the side of length *c*.

Example 1

The size of a rectangular television screen is given as the length of the diagonal of the screen. If the length of the screen is 24 in. and the width is 18 in., what is the diagonal length?

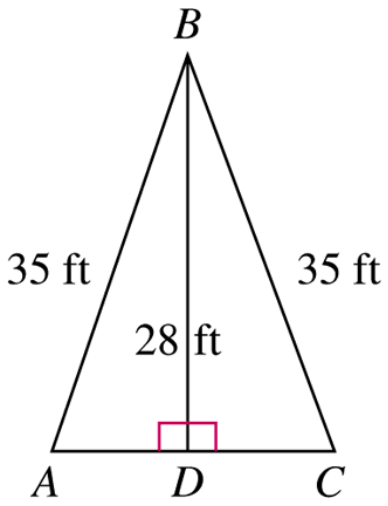


$$c^2 = 18^2 + 24^2 \Rightarrow c^2 = 900 \Rightarrow c = 30$$

(To find c, take the square root of both sides of the equation.)

Example 2

A pole, $\overline{BD}$, 28 ft high, is perpendicular to the ground. Two wires, $\overline{BC}$ and $\overline{BA}$, each 35 ft long, are attached to the top of the pole and to stakes *A* and *C* on the ground. If points *A*, *D*, and *C* are collinear, how far are the stakes *A* and *C* from each other?



$28^2 + (DC)^2 = 35^2$

$784 + (DC)^2 = 1225$

$(DC)^2 = 441$

$(DC) = 21$

So, AC = 2 x 21 = 42 ft.

Example 3

Determine if the following can be the lengths of the sides of a right triangle:

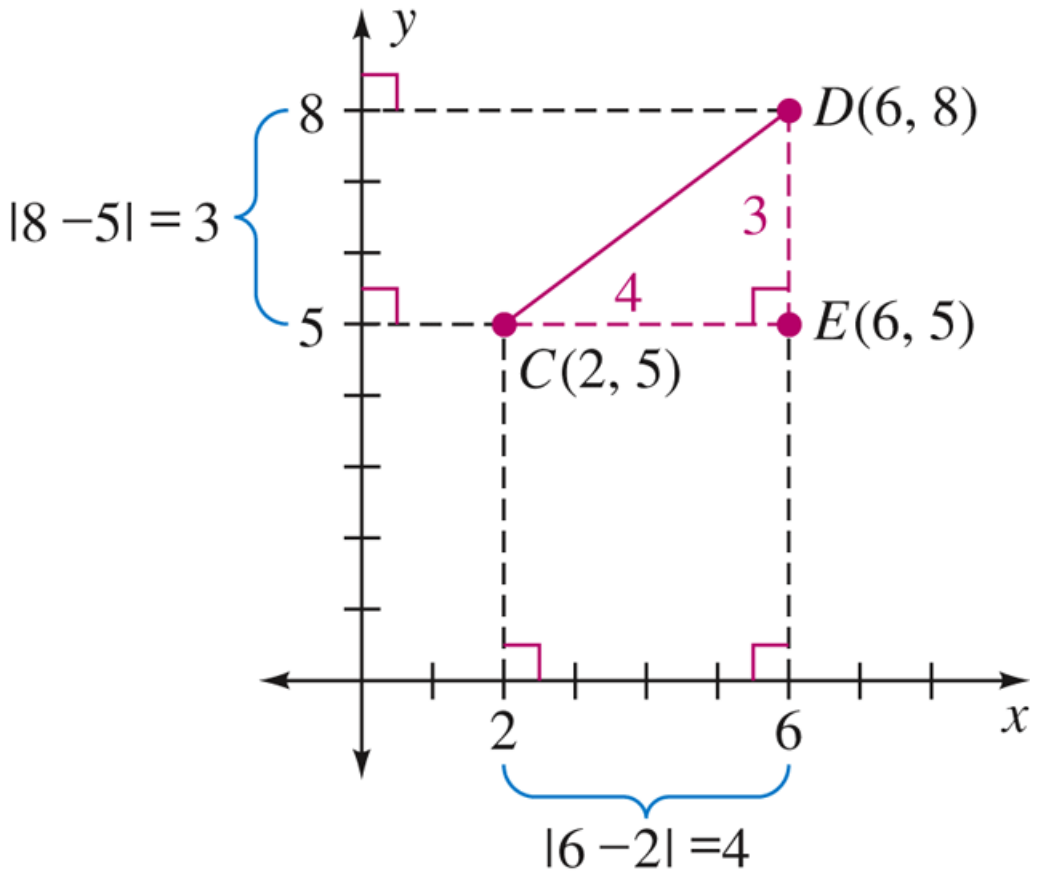
a. 51, 68, 85 b. 2, 3, $\sqrt{13}$ c. 3, 4, 7

$51^2 + 68^2 = 85^2$ $2^2 + 3^2 = \left(\sqrt{13}\right)^2$ $3^2 + 4^2 \neq 7^2$

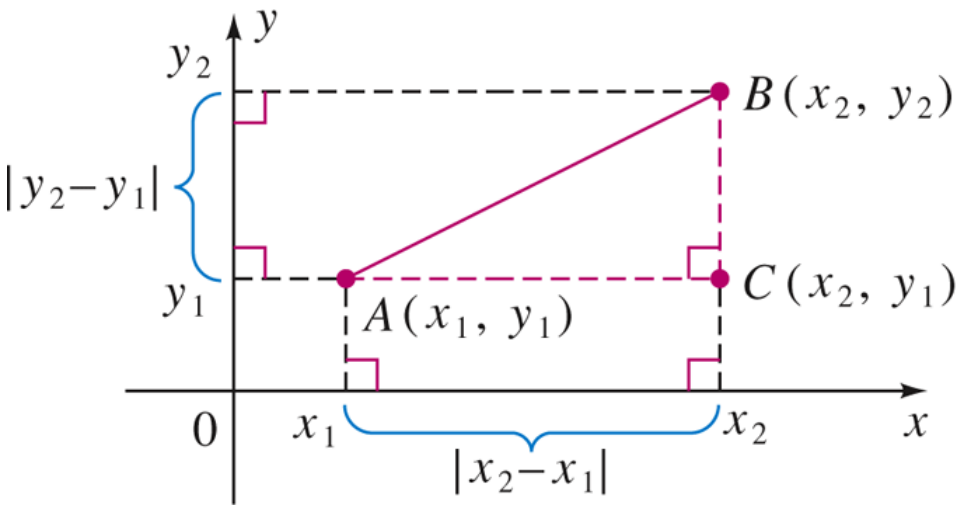
yes yes no

The Distance Formula: An Application of the Pythagorean Theorem

$$CD^2 = DE^2 + CE^2$$
$$= 3^2 + 4^2$$
$$= 5^2 = 25$$
$$CD = 5$$



The Distance Formula



The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 4 (1 of 2)

Show that $A(7,4), B(^{-}2,1), \text{and } C(10,^{-}5)$ are the vertices of an isosceles triangle. Then show that ΔABC is a right triangle.

$$AB = \sqrt{\left(^{-}2-7\right)^2 + (1-4)^2} = \sqrt{90}$$

$$BC = \sqrt{\left[10-\left(^{-}2\right)\right]^2 + \left(^{-}5-1\right)^2} = \sqrt{180}$$

$$AC = \sqrt{(10-7)^2 + \left(^{-}5-4\right)^2} = \sqrt{90}$$

$AB = AC$, so the triangle is isosceles.

Example 4 (2 of 2)

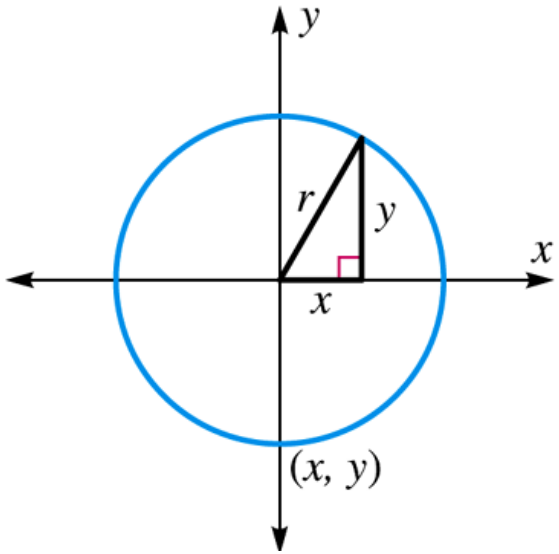
$$\left(\sqrt{90}\right)^2+\left(\sqrt{90}\right)^2=\left(\sqrt{180}\right)^2$$

ΔABC is a right triangle with hypotenuse $\overline{BC}$.

Using the Distance Formula to Develop the Equation of a Circle (1 of 2)

From the distance formula, we have

$$r = \sqrt{x^2 + y^2}.$$

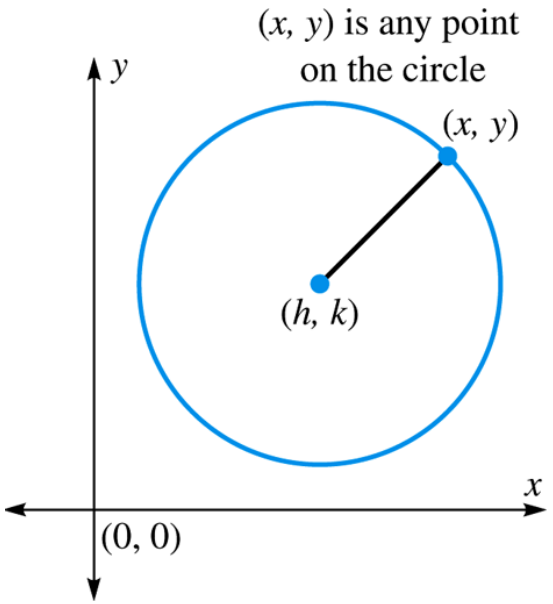


The equation of a circle with the center at the origin and radius r is $r^2 = x^2 + y^2$.

Using the Distance Formula to Develop the Equation of a Circle (2 of 2)

From the distance formula, we have

$$r = \sqrt{(x-h)^2 + (y-k)^2}.$$



The equation of a circle with the center (h,k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.

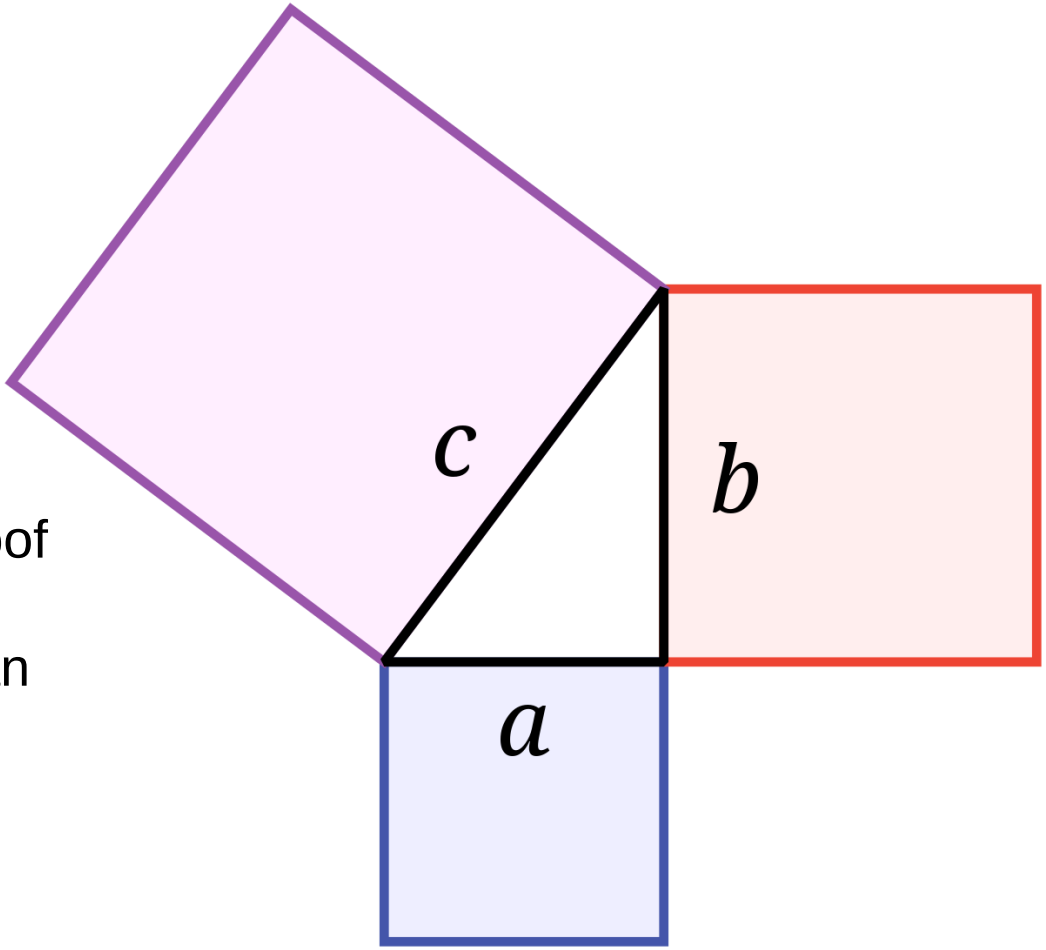
Interesting Facts About Pythagoras

- He ran a “school” that was pretty much a cult
- Once you joined his cult, you couldn’t meet Pythagoras until after a 5 year initiation period where you were required to not speak.
- Members of the cult could not eat beans because the Pythagoreans believe that beans and humans were spawn from the same thing and so eating beans was the same as eating human flesh.

Interesting Facts About Pythagoras

- Members had tons of rules to follow like you HAD to put on your right sandal before the left
- If you broke the rules, you were thrown out and a tombstone was erected to mourn your exile and death from the cult
- Very progressive in his thinking that women should be taught mathematics and philosophy
- Probably isn’t solely responsible for the findings that are attributed to him

That ends section 13-3.



A visual proof of the Pythagorean Theorem.