Section 12-4 Similar Triangles

Students will be able to understand and explain

- Similar figures.
- Theorems for determining similarity for triangles.
- Properties of proportion for similar triangles.
- Midsegments of triangles.
- Indirect measurements using similar triangles.

Definition of Similar Triangles

ΔABC is similar to ΔDEF, written $\Delta ABC \sim \Delta DEF$,

if, and only if, the corresponding interior angles are congruent and the lengths of the corresponding sides are proportional; that is,

$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$, and

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}.$$

The common ratio of lengths is the **scale factor**.

Similar Triangle Theorems (1 of 2)

SSS Similarity for Triangles

If the lengths of corresponding sides of two triangles are proportional, then the triangles are similar.

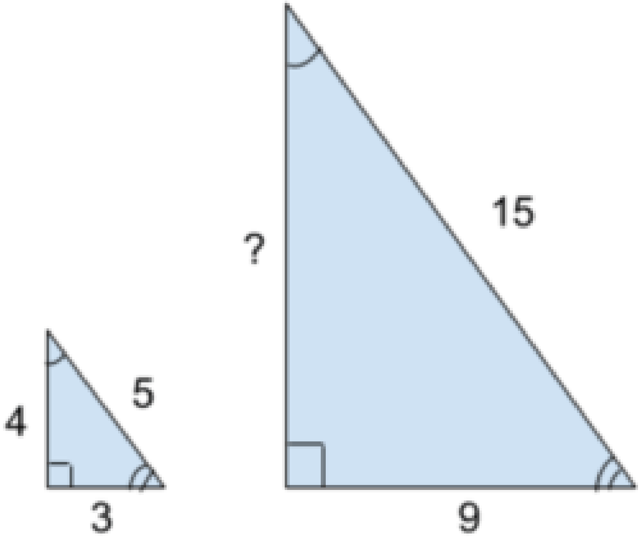
SAS Similarity for Triangles

If in two triangles two sides of one triangle are proportional to their corresponding sides in the other triangle and the included angles are congruent, then the triangles are similar.

Similar Triangle Theorems (2 of 2)

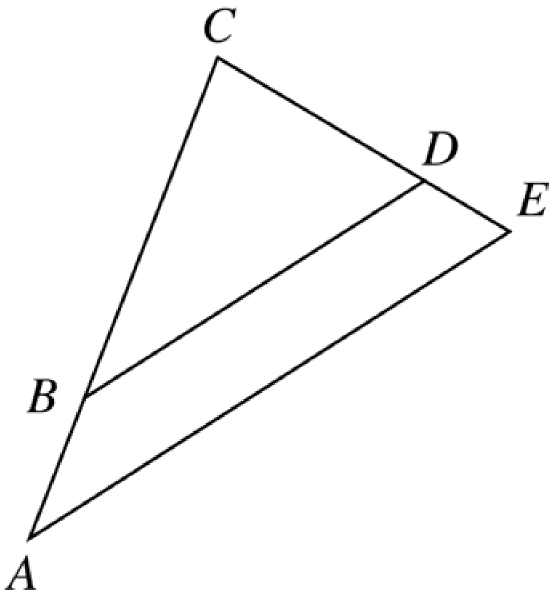
Angle, Angle (AA) Similarity for Triangles

If two angles of one triangle are congruent, respectively, to two angles of a second triangle, then the triangles are similar.



Example 1

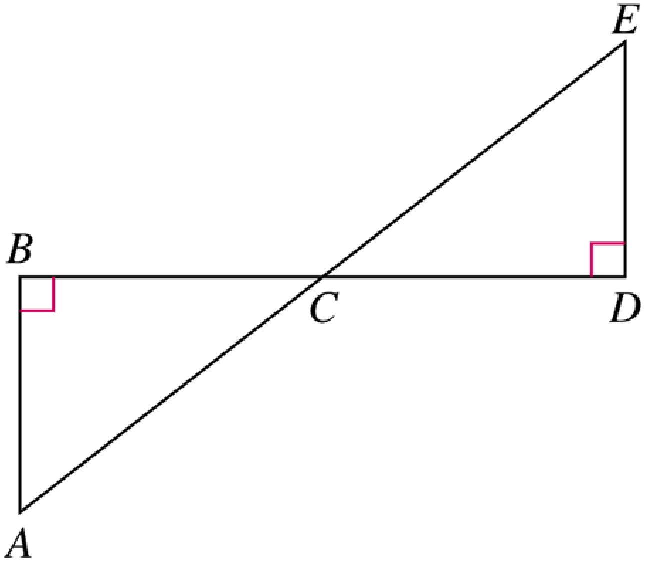
For each figure, find a pair of similar triangles.



$\overline{AE} \parallel \overline{BD}$

Because $\overline{AE} \parallel \overline{BD}$, congruent corresponding angles are formed by a transversal cutting the parallel segments. Thus, $\angle CBD \cong \angle CAE$ and $\angle CDB \cong \angle CEA$. Also, $\angle C \cong \angle C$, so $\Delta CBD \sim \Delta CAE$ by AA.

Example 2

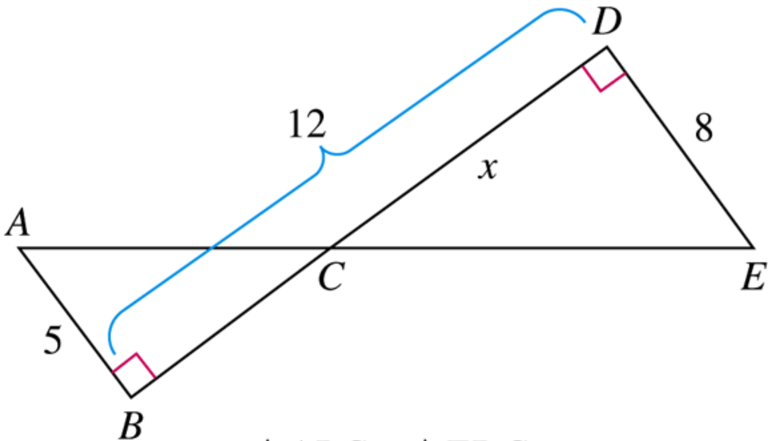


$\overline{BD}$ intersects $\overline{AE}$ in C

$\angle B \cong \angle D$ because both are right triangles. Also, $\angle ACB \cong \angle ECD$ because they are vertical angles. Therefore, $\Delta ACB \sim \Delta ECD$ by AA.

Example 3

In the figure, solve for *x*.



$\triangle ABC \sim \triangle EDC$

Since $\Delta ABC \sim \Delta EDC$,

$$\frac{AB}{ED} = \frac{AC}{EC} = \frac{BC}{DC}.$$

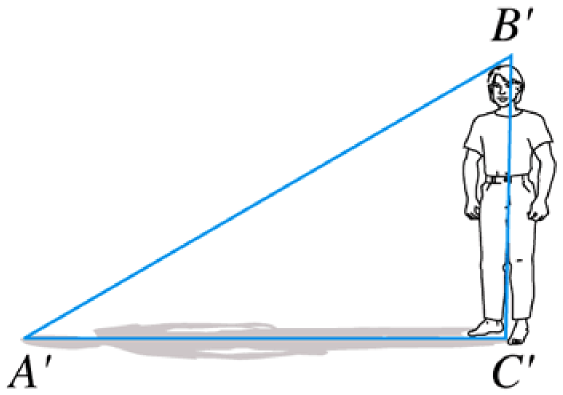
$AB = 5,$ $ED = 8,$ and $CD = x,$ so $BC = 12 - x.$ So,

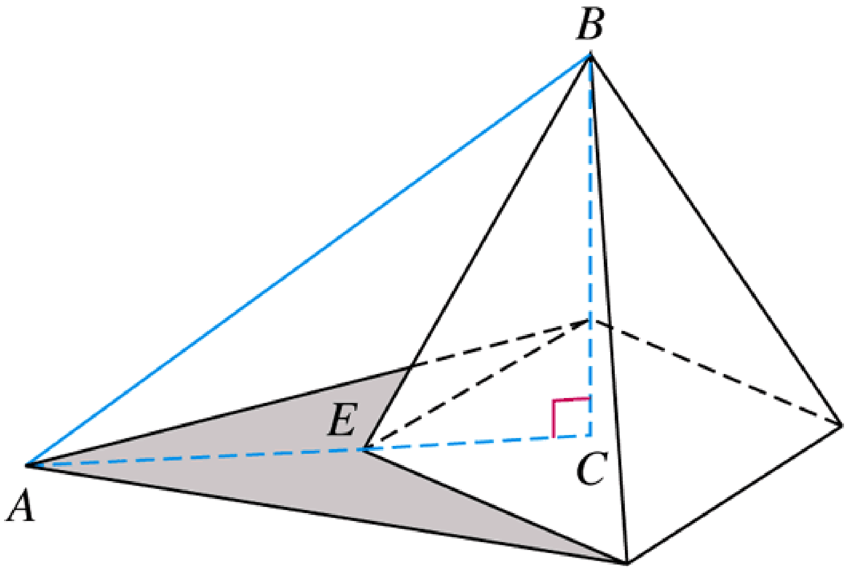
$$\frac{5}{8} = \frac{12 - x}{x} \Rightarrow 5x = 8(12 - x) \Rightarrow 5x = 96 - 8x \Rightarrow$$

$$13x = 96 \Rightarrow x = \frac{96}{13}$$

Indirect Measurements

Similar triangles have long been used to make indirect measurements.

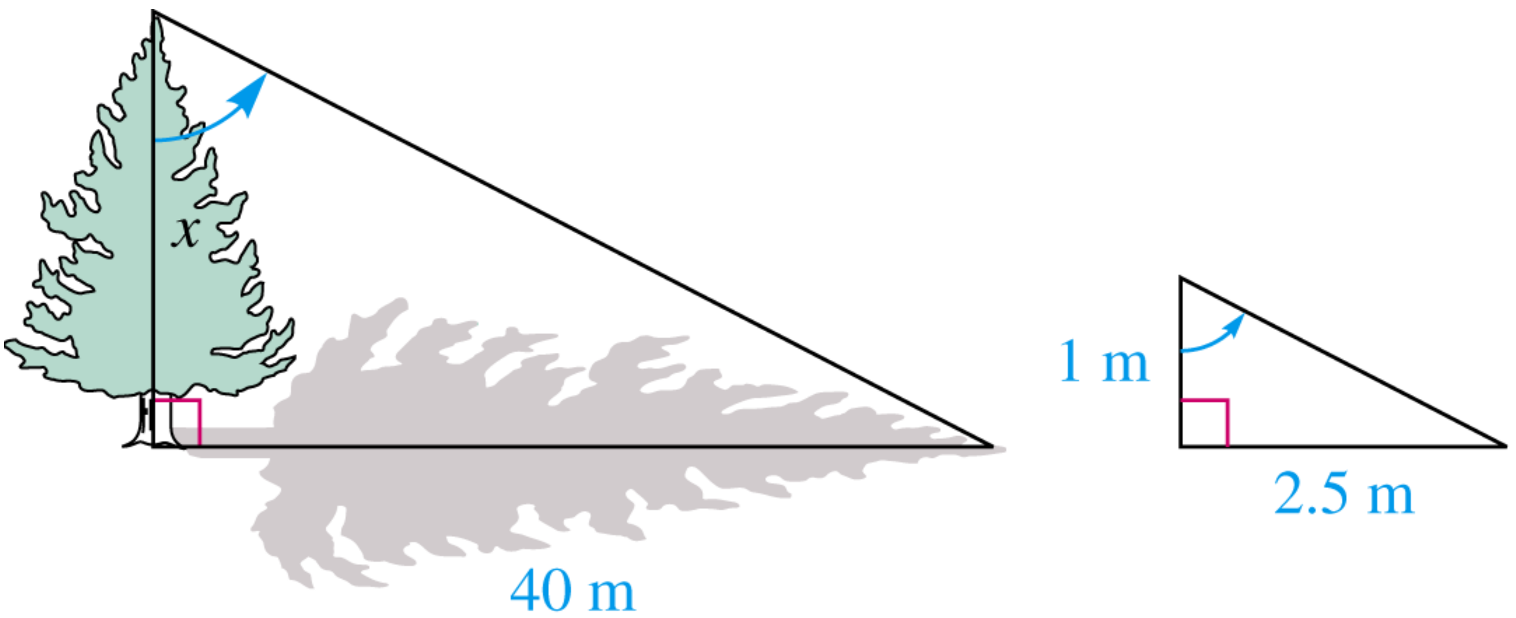




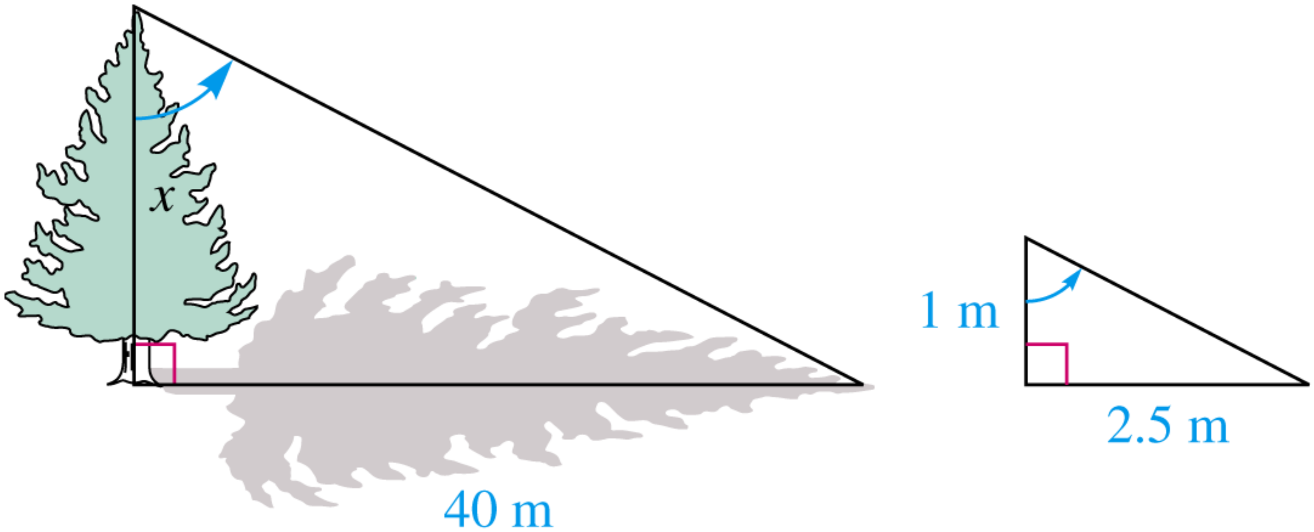
We can determine the height of the pyramid using similar triangles.

Example 4 (1 of 2)

On a sunny day, a tall tree casts a 40-m shadow. At the same time, a meterstick held vertically casts a 2.5-m shadow. How tall is the tree?



Example 4 (2 of 2)



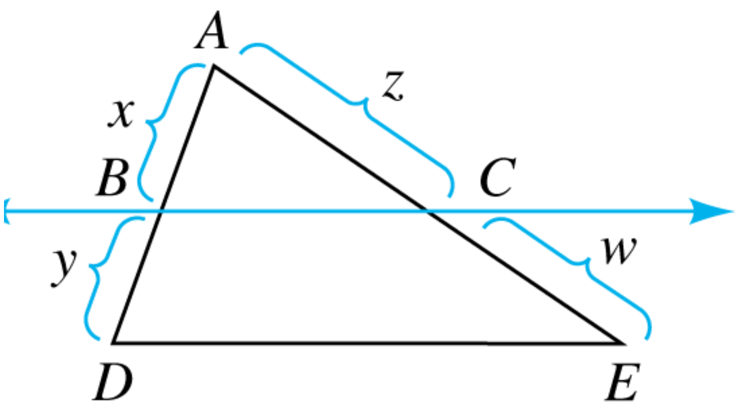
The triangles are similar, so

$$\frac{x}{40} = \frac{1}{2.5} \Rightarrow 2.5x = 40 \Rightarrow x = 16$$

The tree is 16 meters tall.

Properties of Proportion (1 of 3)

If a line parallel to one side of a triangle intersects the other sides, then it divides those sides into proportional segments.



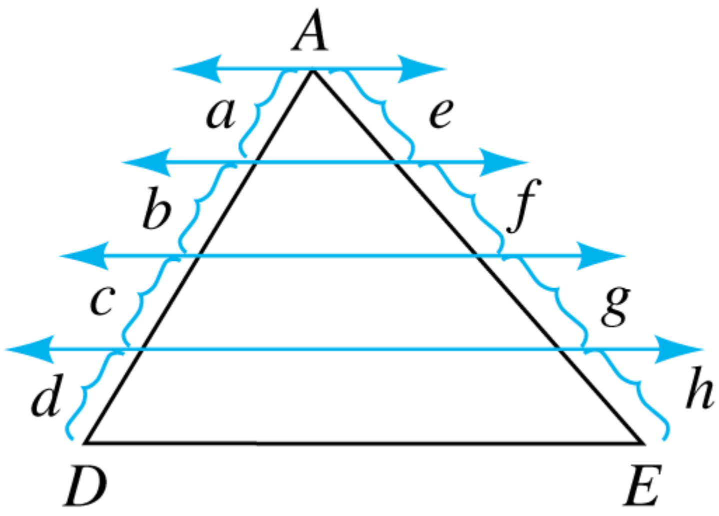
$$\frac{y}{x} = \frac{w}{z}$$

$$\frac{x}{y} = \frac{z}{w}$$

If a line divides two sides of a triangle into proportional segments, then the line is parallel to the third side.

Properties of Proportion (2 of 3)

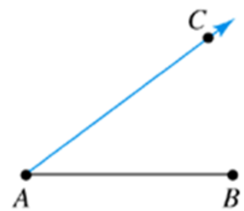
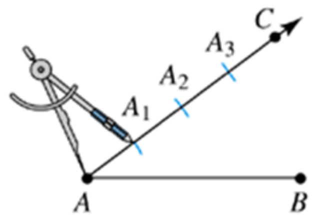
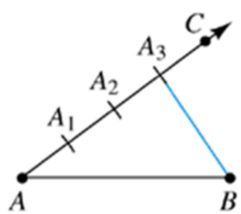
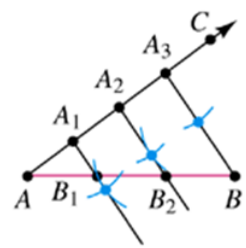
If parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on any transversal.



If parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on any transversal.

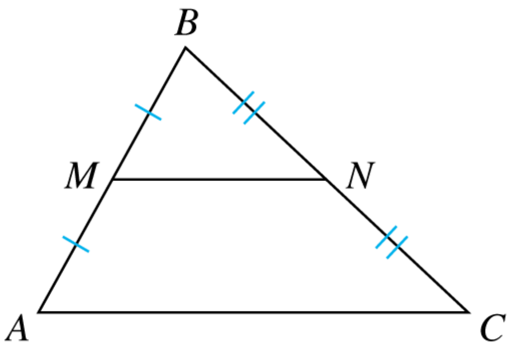
Properties of Proportion (3 of 3)

Using only a compass and a straightedge, we can divide segment $\overline{AB}$ into three congruent parts.

				
To separate $\overline{AB}$ into a given number of congruent parts:	Draw any ray, $\overrightarrow{AC}$, such that *A*, *B*, and *C* are noncollinear.	Mark off the given number of congruent segments (of any size) on $\overrightarrow{AC}$. In this case, we use three congruent segments.	Connect *B* to A_3.	Through A_2 and A_1, construct parallels to $\overline{BA_3}$. Thus, $\overline{AB_1} \cong \overline{B_1B_2} \cong \overline{B_2B}$.

Midsegments of Triangles and Quadrilaterals

The **midsegment** (segment connecting the midpoints of two sides of a triangle) is parallel to the third side of the triangle and half as long.



If a line bisects one side of a triangle and is parallel to a second side, then it bisects the third side and therefore is a midsegment.

Link to video construction:

- Separating a line segment into 3 congruent parts (2:21) https://www.youtube.com/watch?v=NEFjuJjzgtg

- That ends section 12-4