Section 13-5 Volume, Mass, and Temperature

Students will be able to understand and explain

- Volumes or prisms, cylinders, pyramids, cones, and spheres.
- Converting metric measures of volume.
- Converting English measures of volume.
- Ratios of volume of similar figures.
- Measures of mass and capacity.

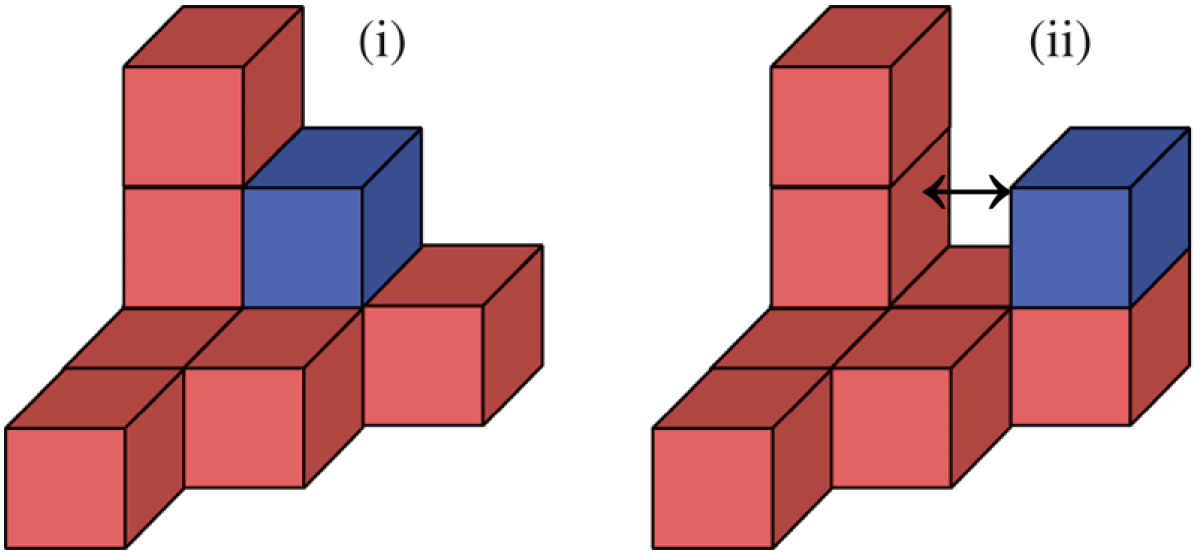
Surface Area vs Volume

- Surface Area and Volume are often confused.
- They both deal with 3-dimensional figures, however:
  - Surface Area is the number of squares needed to cover an object
  - Volume is the number of cubes needed to build an object
  - The volume of an object is the amount of space that object takes up.

Volume and Surface Area

The volume of each figure is 9 cubic units

What is the surface area of each figure?



34 square units

36 square units

By moving the blue cube, we expose 2 square sides that were not visible in figure (i).

English Measures of Volume

Basic Units of Volume:

cubic inch (in.^3)

cubic foot (ft^3)

cubic yard (yd^3)

Example 1 (1 of 2)

Convert each of the following:

a. $45\text{ yd}^3 = \underline{\qquad\qquad\qquad\qquad}\text{ ft}^3$

$$45\text{ yd}^3 = 45\text{ yd}^3 \cdot \frac{(3\text{ ft})^3}{1\text{ yd}^3} = 45\text{ yd}^3 \cdot \frac{27\text{ ft}^3}{1\text{ yd}^3} = 1215\text{ ft}^3$$

b. $4320\text{ in.}^3 = \underline{\qquad\qquad\qquad\qquad}\text{ yd}^3$

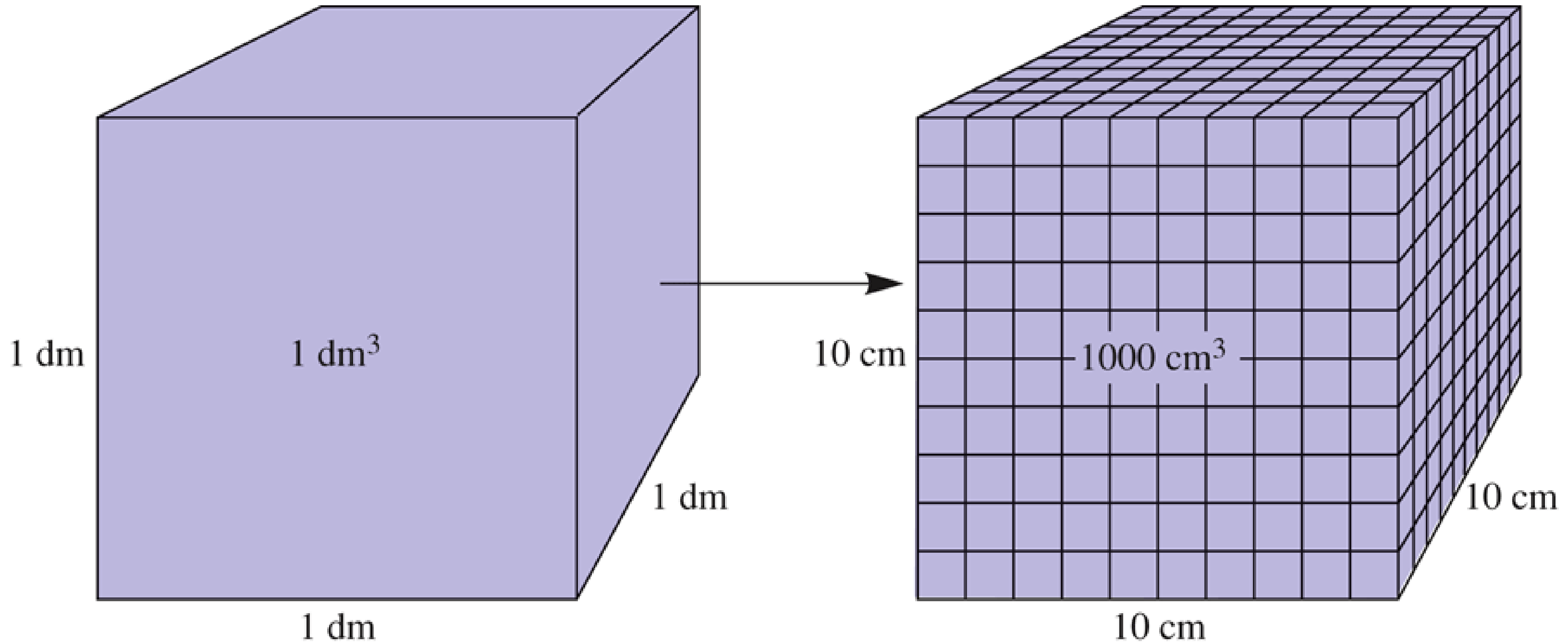
$$4320\text{ in.}^3 = 4320\text{ in.}^3 \cdot \frac{1\ yd^3}{36^3\text{ in}^3} \approx 0.0926\text{ yd}^3 \approx 0.1\text{ yd}^3$$

Example 1 (2 of 2)

c. $3\text{ ft}^3 = \underline{\hspace{10em}}\ \text{yd}^3$

$$3\text{ ft}^3 \cdot \left(\frac{1\text{ yd}^3}{3^3\ \text{ft}^3}\right) = \frac{3}{27}\text{yd}^3 \approx 0.1\text{ yd}^3$$

Converting Metric Measures of Volume



$$1 \text{ dm}^3 = 1000 \text{ cm}^3$$

Recall in 13-2 when we converted metric units of area, we used the stairsteps and doubled the number of place values moved. To convert metric units of volume, we triple the number of place values moved.

Example 2

Convert each of the following:

a. $5\ \text{m}^3 = \underline{\quad 5{,}000{,}000 \quad}\ \text{cm}^3$

b. $12{,}300\ \text{mm}^3 = \underline{\quad 12.3 \quad}\ \text{cm}^3$

km hm dam m dm cm mm

Volume vs Capacity

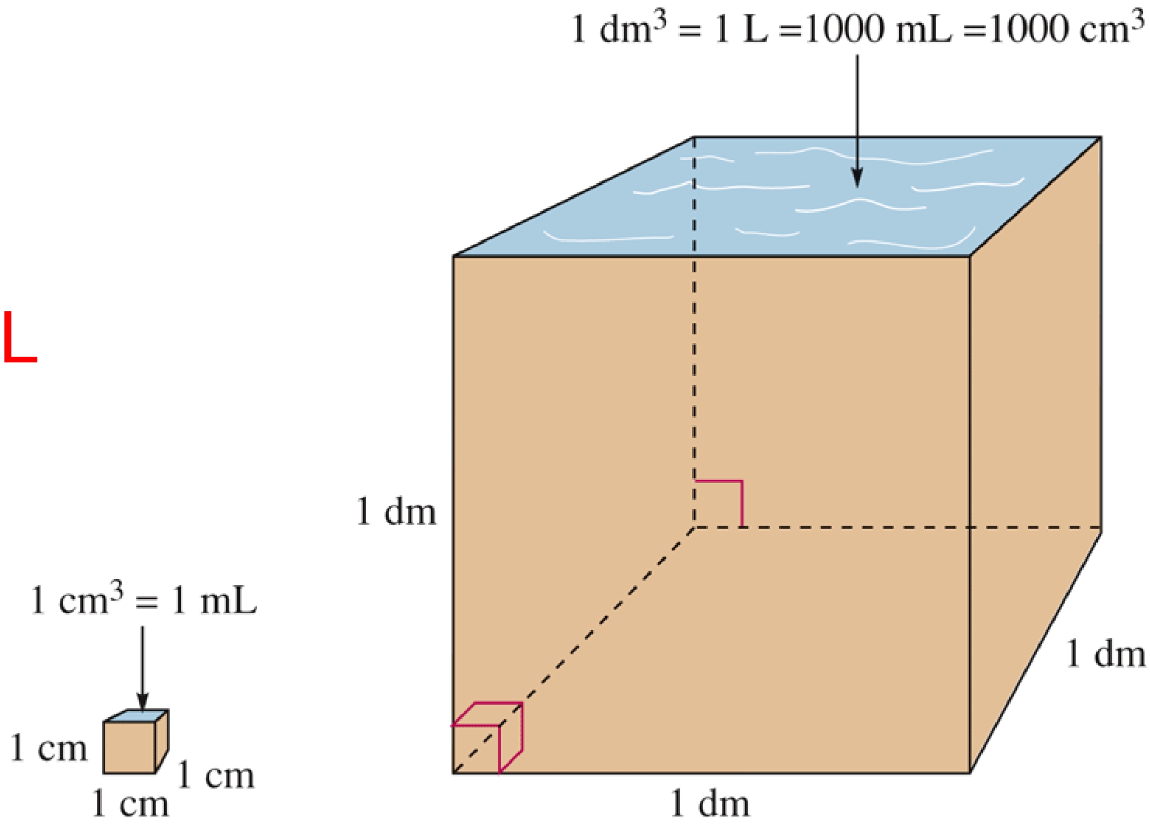
- The volume of an object is the amount of space that object takes up.
- The capacity of an object is the amount of liquid needed to fill an object.
- We typically use cubic units to measure the volume and units like mL, gallons, quart, etc to measure capacity.

Converting Metric Measures of Volume

$1\ cm^3 = 1\ cc$

$1\ mL = 1\ cm^3$

$1000\ mL = 1\ L$



$$1\ \text{liter} = 1\ dm^3 = 1000\ cm^3$$

Metric Units of Capacity

Unit	Symbol	Relation to Liter
kiloliter	kL	1000 L
hectoliter	hL	100 L
dekaliter	daL	10 L
liter	**L**	**1 L**
deciliter	dL	0.1 L
centiliter	cL	0.01 L
milliliter	mL	0.001 L

Example 3

Convert each of the following:

a. 27 L = ____27,000____ mL

b. 362 mL = ____0.362____ L

c. 3 mL = ____3____ cm^3

d. 3 m^3 = ____3000____ L

These are not cubic units, so we do not triple the number of place values moved.

English Units of Capacity

Units of Capacity & Volume

English System

1 pint (pt) = 16 fluid ounces (oz)

1 quart (qt) = 2 pints (pt)

1 gallon (gal) = 4 quarts (qt)

Volume vs. Capacity

1 cubic foot ≈ 7.48 gal

1 cubic yard ≈ 202 gal

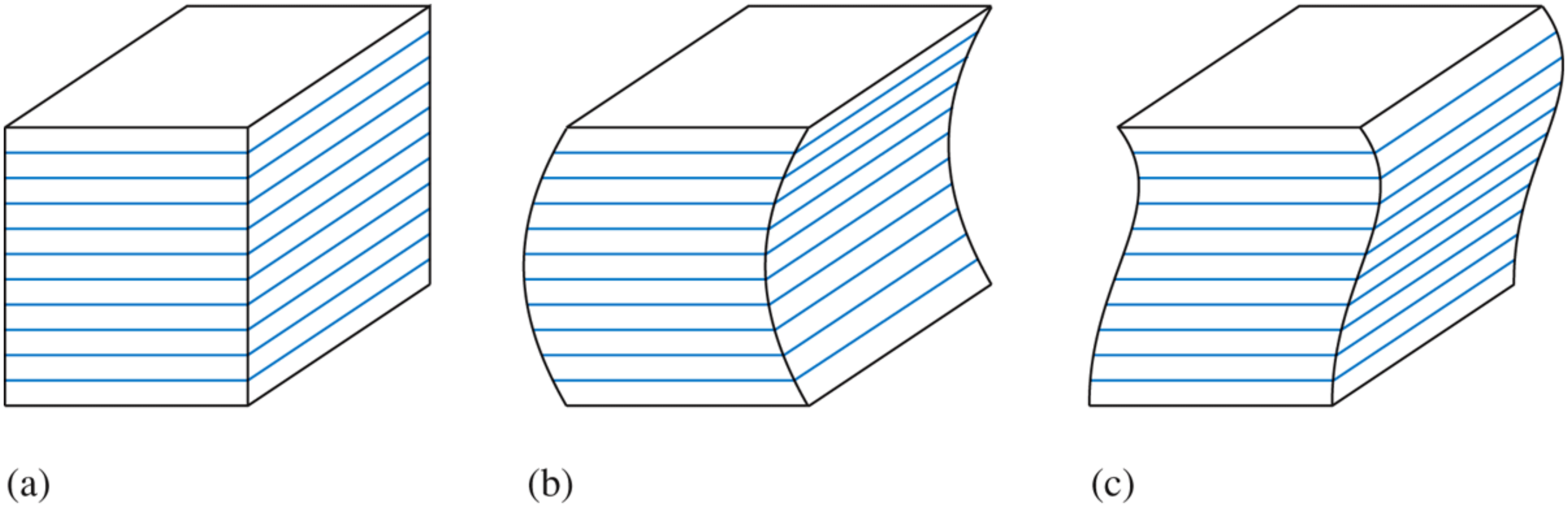
1 gal. ≈ 231 in.^{3}

1 ft^3 freshwater ≈ 62.5 lb

1 ft^3 seawater ≈ 64 lb

To convert between English and Metric:
1 gallon = 3.8 L

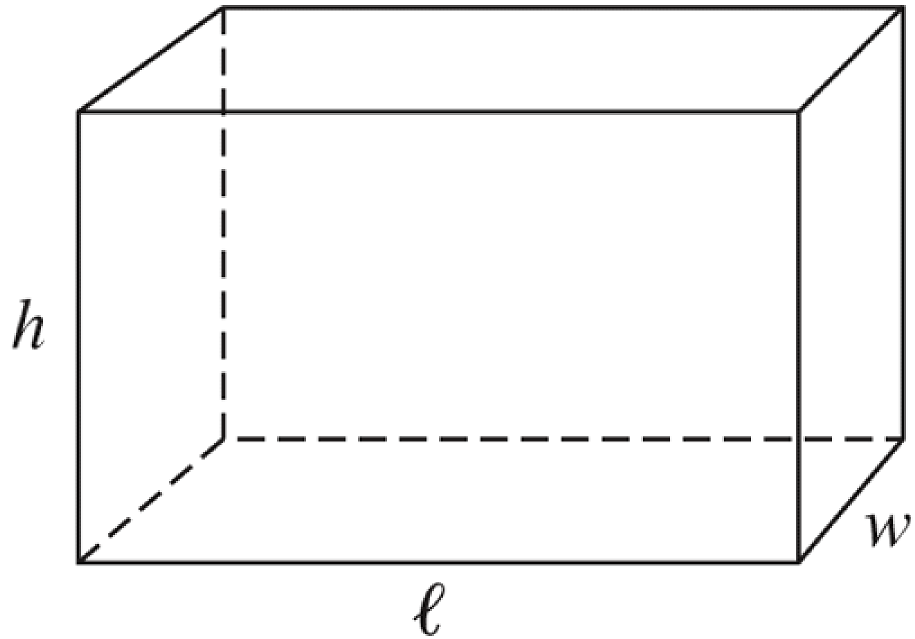
Volumes of Prisms and Cylinders



Cavalieri's Principle

Two solids each with a base in the same plane have equal volumes if every plane parallel to the bases intersects the solids in cross sections of equal area.

Volume of Right Rectangular Prisms



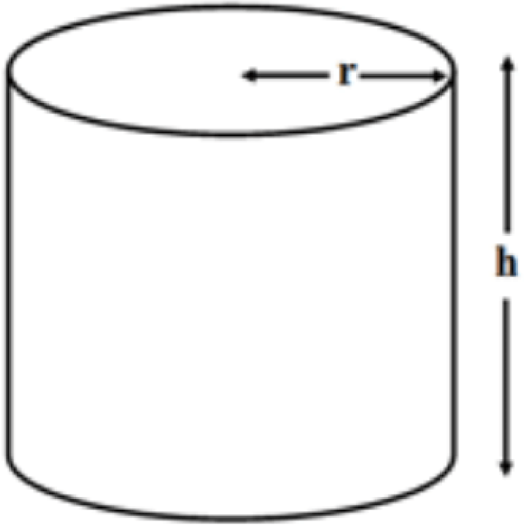
Right rectangular prism

$$\text{Volume} = lwh$$

Note: In the case of a cube, then l, w, and h are all the same length, call it s.
So, volume: $V=s^3$

Volume of a Right Circular Cylinder

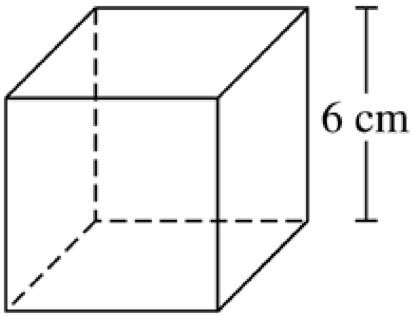
$$V = \pi r^2 h$$



Notice that with prisms and cylinders, the volume is just the area of the base times the height.

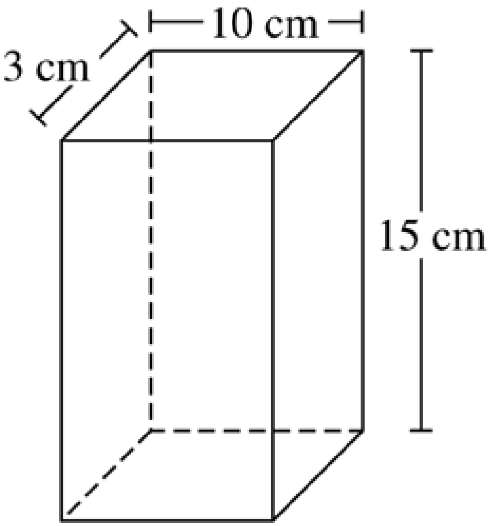
Example 4

Find the volume of each figure.



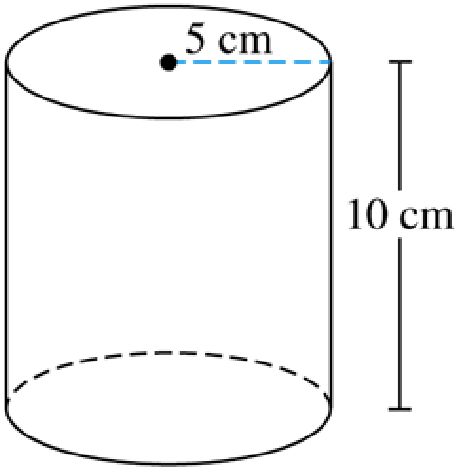
Cube

$6^3 = 216 \text{ cm}^3$



Right rectangular prism

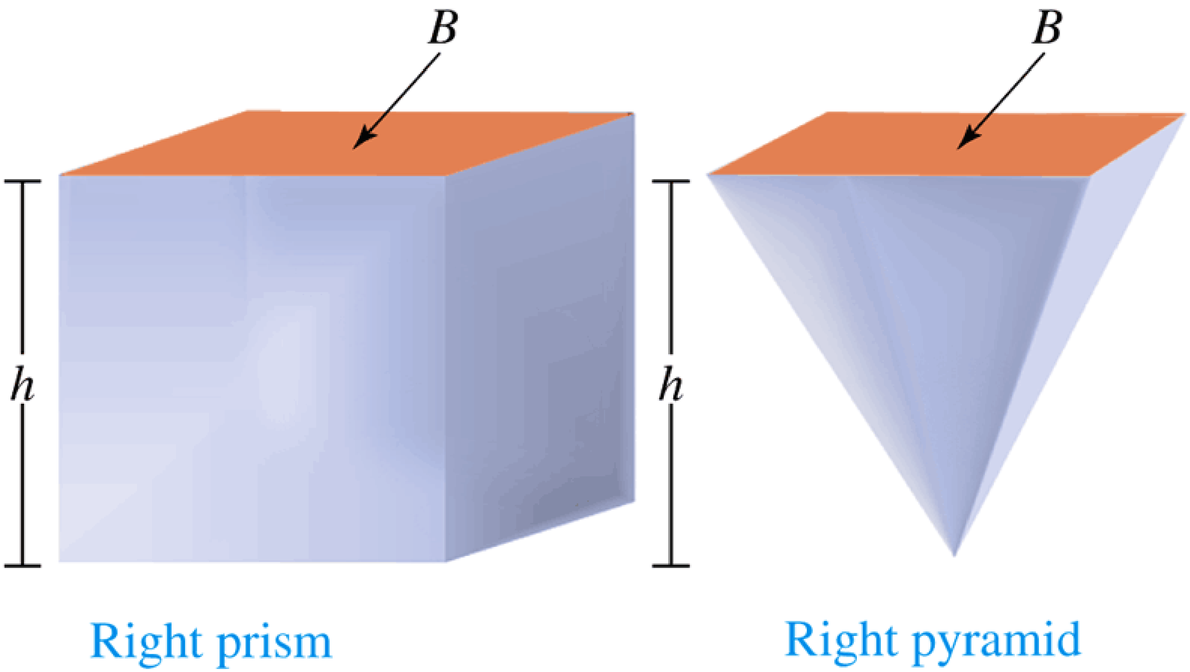
$3 \cdot 10 \cdot 15 = 450 \text{ cm}^3$



Right circular cylinder

$\pi \cdot 5^2 \cdot 10$
$= 250\pi \text{ cm}^3$

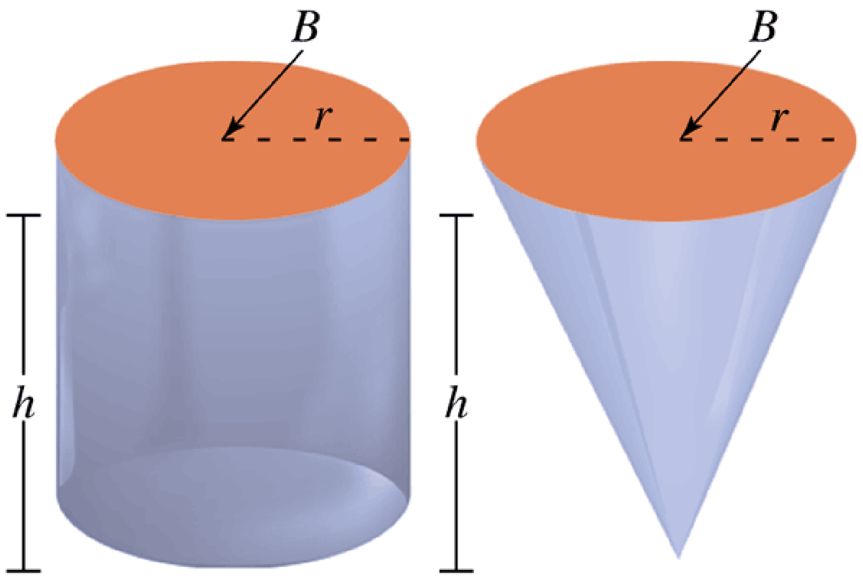
Volume of Pyramids



Right pyramid

$\text{Volume} = \frac{1}{3}Ah$ where *A* is the area of the base.

Volumes of Cones



Right circular cylinder

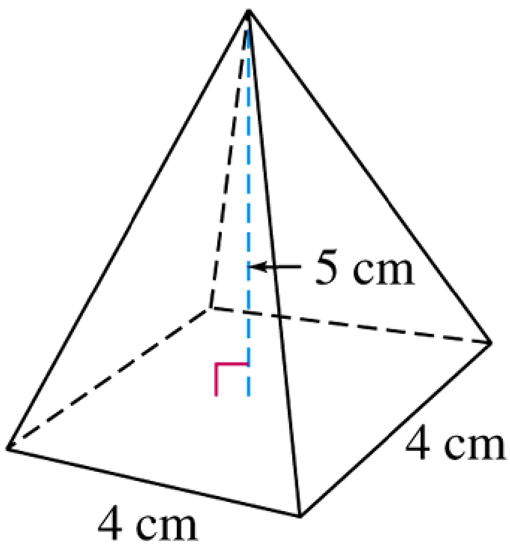
Right circular cone

Right circular cone

$$\text{Volume} = \frac{1}{3}Ah = \frac{1}{3}\pi r^2 h$$

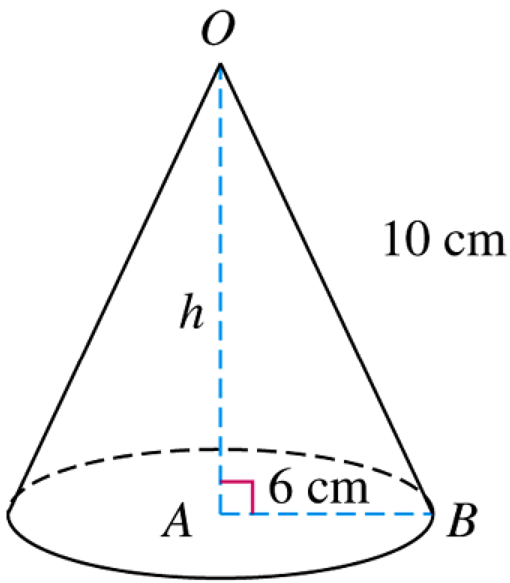
Example 5

Find the volume of each figure.



Right square pyramid

$$\frac{80}{3}\text{cm}^3$$

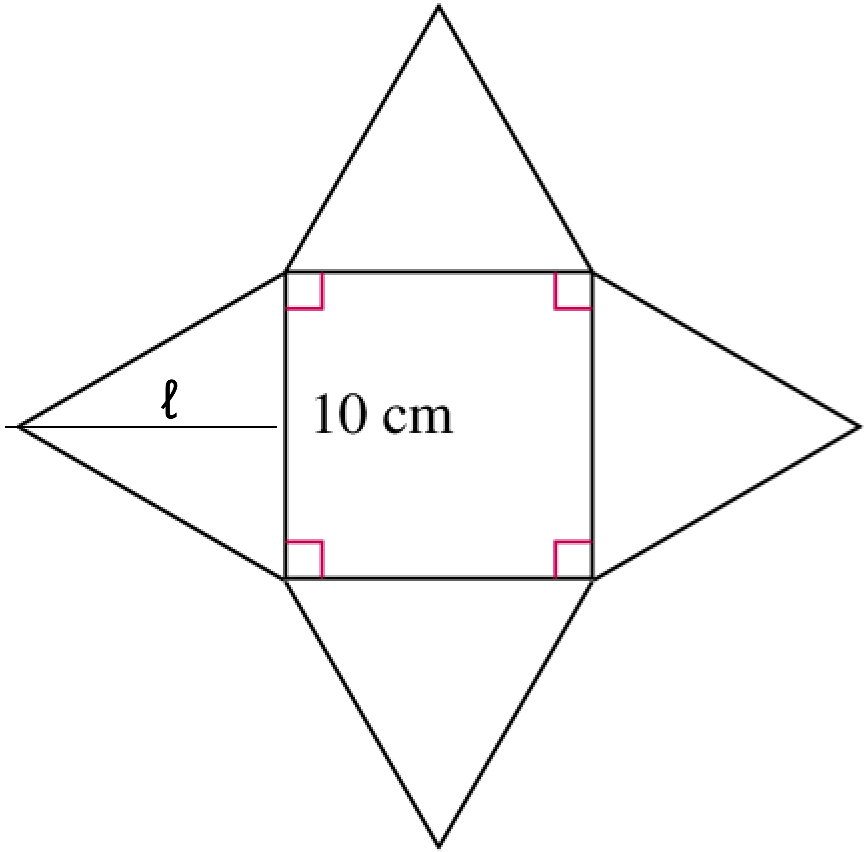


Right circular cone

$$96\pi \text{ cm}^3$$

Example 6

Find the volume of the pyramid represented by the net shown. Each triangle is equilateral.



$$V = \frac{1}{3} \cdot 10^2 \cdot \sqrt{50} \approx 235.7 \text{ cm}^3$$

Where did $\sqrt{50}$ come from?

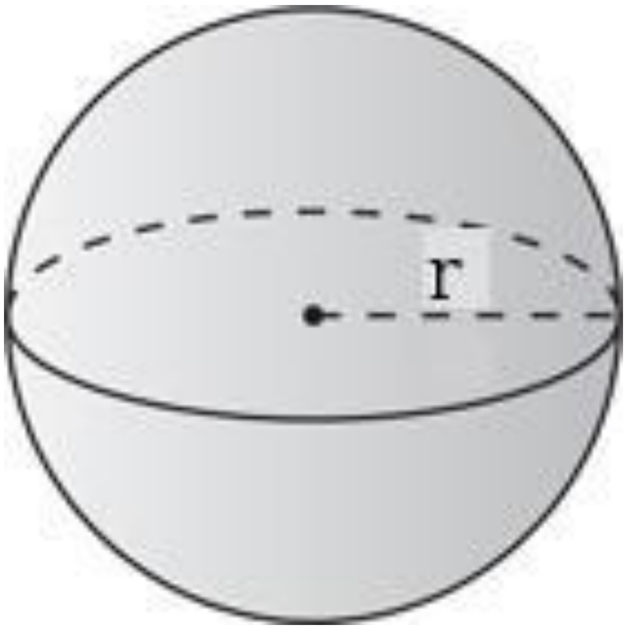
Use the Pythagorean Theorem to find the slant height.

$10^2 = \ell^2 + 5^2$ So, $\ell = 5\sqrt{3}$

Then use the Pythagorean Theorem again to find h.

$\left(5\sqrt{3}\right)^2 = 5^2 + h^2$ So, $h = \sqrt{50}$

Volume of a Sphere



Sphere

$$\text{Volume} = \frac{4}{3}\pi r^3$$

Example 7

Find the volume of a sphere whose radius is 6 cm.

$$V = \frac{4}{3}\pi\left(6\,\text{cm}\right)^3 = 288\pi\ \text{cm}^3$$

Comparing Measurements of Similar Figures Theorems (1 of 2)

The ratio of any linear measurement of two similar figures, for example, length, width, height, perimeter, diagonal, diameter, slant height, have the same scale factor k.

For similar triangles with scale factor k, the ratio of their areas is k^2.

For similar polygons with scale factor k, the ratio of their areas is k^2.

Comparing Measurements of Similar Figures Theorems (2 of 2)

If the scale factor of two similar figures is k, then the scale factor of the areas or surface areas is k^2, and the ratio of volumes is k^3.

Example 8

How does the surface area of a sphere 10 in. in diameter compare with the surface area of a sphere 5 in. in diameter?

Any two spheres are similar.

The ratios of the diameters is $10:5$ or 2.

Ratio of surface areas is 2^2 or 4.

The 10-in sphere has 4 times the surface area of the 5-inch sphere.

Example 9

How does the volume of a sphere 10 in. in diameter compare with the volume of a sphere 5 in. in diameter?

The ratios of the volumes is 2^3 or 8.

The volume of a 10-in sphere has 8 times the volume of the 5-inch sphere.

Mass

Mass: the amount of matter that makes up an object

Weight: a force exerted by gravitational pull

Gram: fundamental unit of mass in the metric system

Mass and weight are often mistakenly used interchangeably. Weight varies from location to location. For instance, your mass on the moon is the same as it is on Earth; however, your weight on the moon is far less than your weight on Earth since there is no gravitational pull.

At sea level, mass and weight are essentially the same.

Earth's gravity exerts a greater force at lower altitudes. This means you weigh more in New Orleans (10 ft below sea level) than you do in Denver (5280 ft above sea level.)

Metric Unit of Mass is the gram (g)

Unit	**Symbol**	**Relation to Gram**
metric ton	T	1,000,000 g
kilogram	kg	1000 g
hectogram	hg	100 g
dekagram	dag	10 g
gram	**g**	**1 g**
decigram	dg	0.1 g
centigram	cg	0.01 g
milligram	mg	0.001 g

Frames of Reference

- 1 gram is about the mass of a small paperclip
- 1 kilogram is about the mass of 2 loaves of bread
- 4 kilograms is about the mass of a newborn baby
- 1 kg is about 2.2 pounds (lbs)

- Fun Fact: The abbreviation lb for pound is derived from the Latin word Libra (meaning scales.)

Example 10

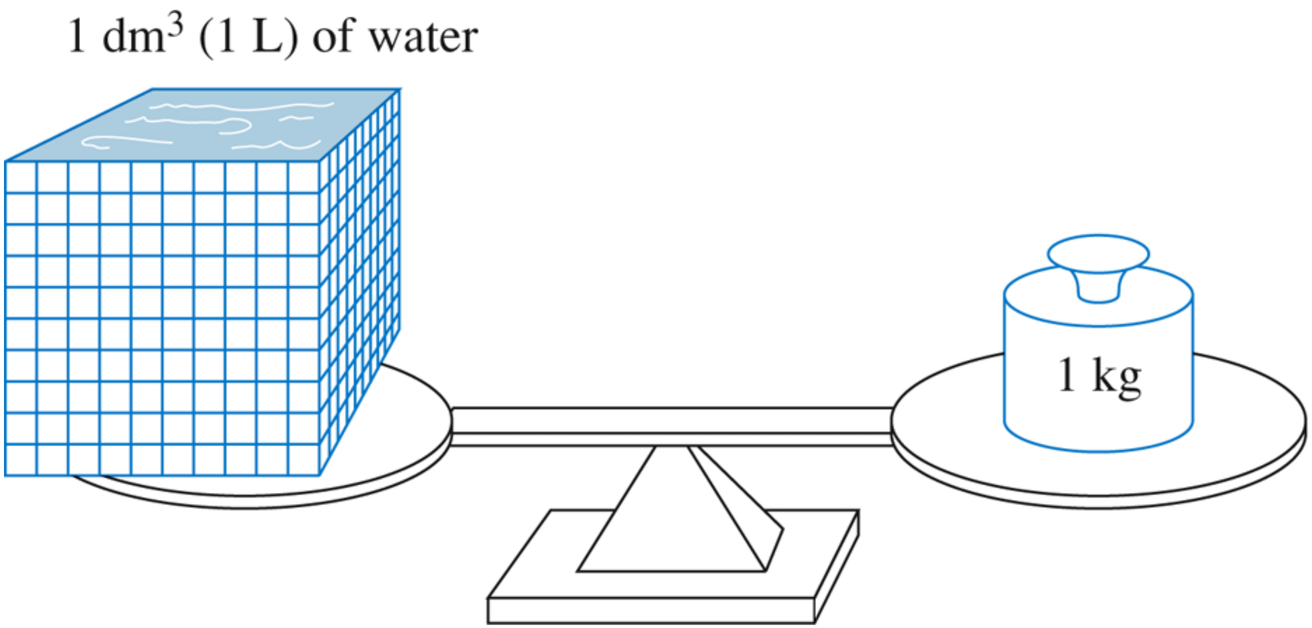
Convert each of the following:

a. 34 g = ____0.034____ kg

b. 6836 kg = ____6.836____ t

Again, notice that since the units are not cubed, we do not triple the number of place values moved.

Relationships Among Metric Units of Volume, Capacity, and Mass



$1\ cm^3$ (1 mL) of water = 1 g

$1\ dm^3$ (1 L) of water = 1 kg

Choose the most realistic measure

- The mass of an iphone: 190kg, 190g, 190mg
- The length of an ant: 2km, 2m, 2mm
- The volume of a wine bottle: 750kL, 750L, 750mL
- The area of an average home: $1500ft^2$, $1500yd^2$, $1500mi^2$
- The area of PSC's campus: $123mi^2$, $123yd^2$, 123 acres
- The length of a city block: 200km, 200m, 200cm
- The volume of a washing machine: $3.2yd^3$, $3.2ft^3$, $3.2in^3$

Choose the most realistic measure (answers)

- The mass of an iphone: 190kg, 190g, 190mg
- The length of an ant: 2km, 2m, 2mm
- The volume of a wine bottle: 750kL, 750L, 750mL
- The area of an average home: $1500ft^2$, $1500yd^2$, $1500mi^2$
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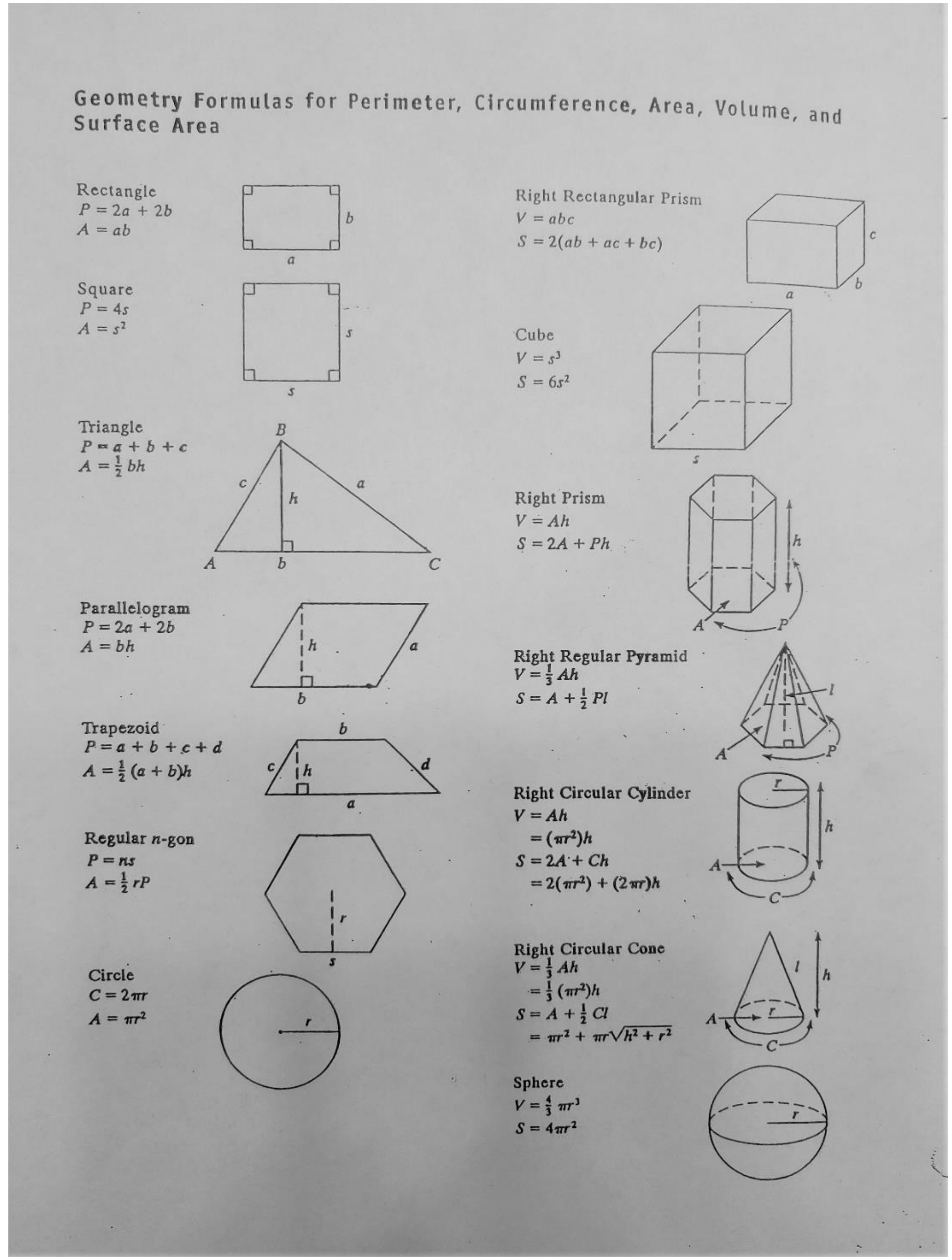
- The length of a city block: 200km, 200m, 200cm
- The volume of a washing machine: $3.2yd^3$, $3.2ft^3$, $3.2in^3$

Just in case you need it again.



Geometry Formulas for Perimeter, Circumference, Area, Volume, and Surface Area

Rectangle
$P = 2a + 2b$
$A = ab$

Square
$P = 4s$
$A = s^2$

Triangle
$P = a + b + c$
$A = \frac{1}{2}bh$

Parallelogram
$P = 2a + 2b$
$A = bh$

Trapezoid
$P = a + b + c + d$
$A = \frac{1}{2}(a + b)h$

Regular n-gon
$P = ns$
$A = \frac{1}{2}rP$

Circle
$C = 2\pi r$
$A = \pi r^2$

Right Rectangular Prism
$V = abc$
$S = 2(ab + ac + bc)$

Cube
$V = s^3$
$S = 6s^2$

Right Prism
$V = Ah$
$S = 2A + Ph$

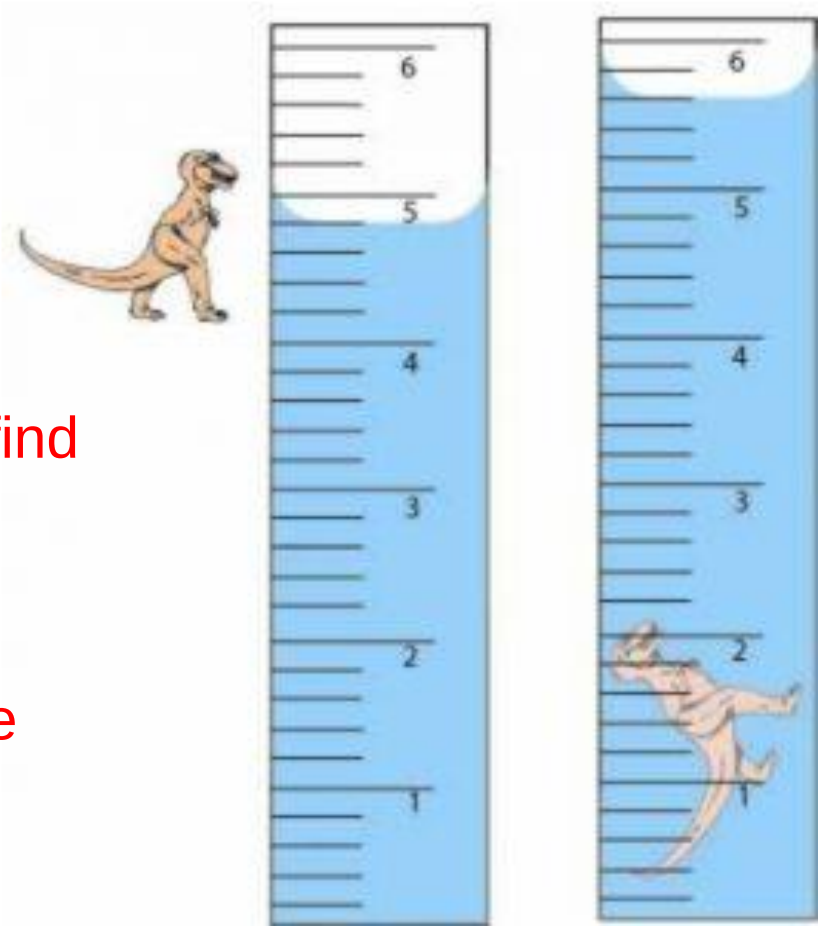
Right Regular Pyramid
$V = \frac{1}{3}Ah$
$S = A + \frac{1}{2}Pl$

Right Circular Cylinder
$V = Ah$
$= (\pi r^2)h$
$S = 2A + Ch$
$= 2(\pi r^2) + (2\pi r)h$

Right Circular Cone
$V = \frac{1}{3}Ah$
$= \frac{1}{3}(\pi r^2)h$
$S = A + \frac{1}{2}Cl$
$= \pi r^2 + \pi r\sqrt{h^2 + r^2}$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

That's it for section 13-5



This shows how to find the volume of an irregular shape by submerging it into water and noting the change in volume.