Section 12-2 Additional Congruence Theorems

Students will be able to understand and explain

- Geometric constructions leading to the ASA and AAS congruence properties.
- Properties of quadrilaterals.

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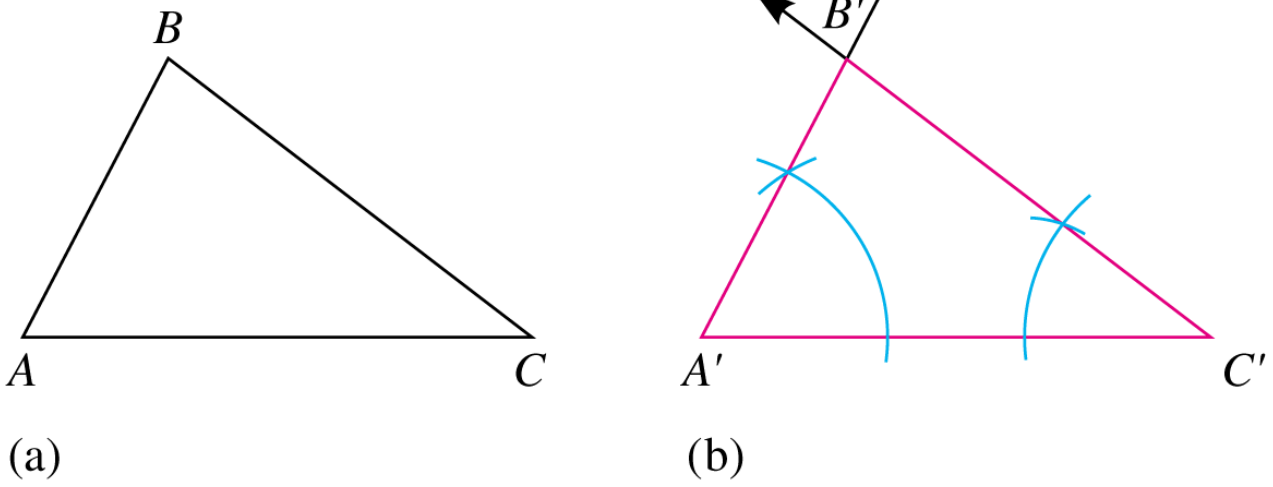
Angle, Side, Angle (ASA) Property

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, respectively, then the triangles are congruent.



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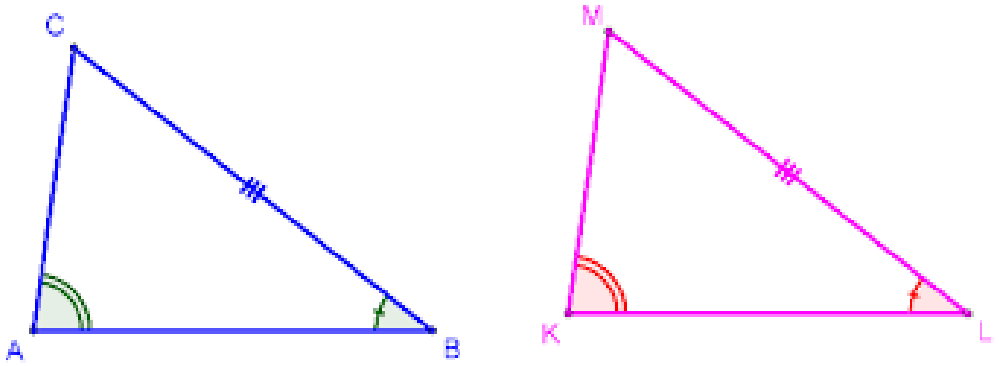
Side, Angle, Angle (SAA) Property

If two angles and the non-included side of one triangle are congruent to the corresponding two angles and non-included side of another triangle, the two triangles are congruent.



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ASA is the same as SAA

- How is ASA the same as SAA?
- We know the sum of the angles in any triangle is 180 degrees. So, if we know that 2 angles are congruent, then we are guaranteed the 3rd angle is also congruent.
- Therefore, ASA can also be written as ASAA or just SAA.

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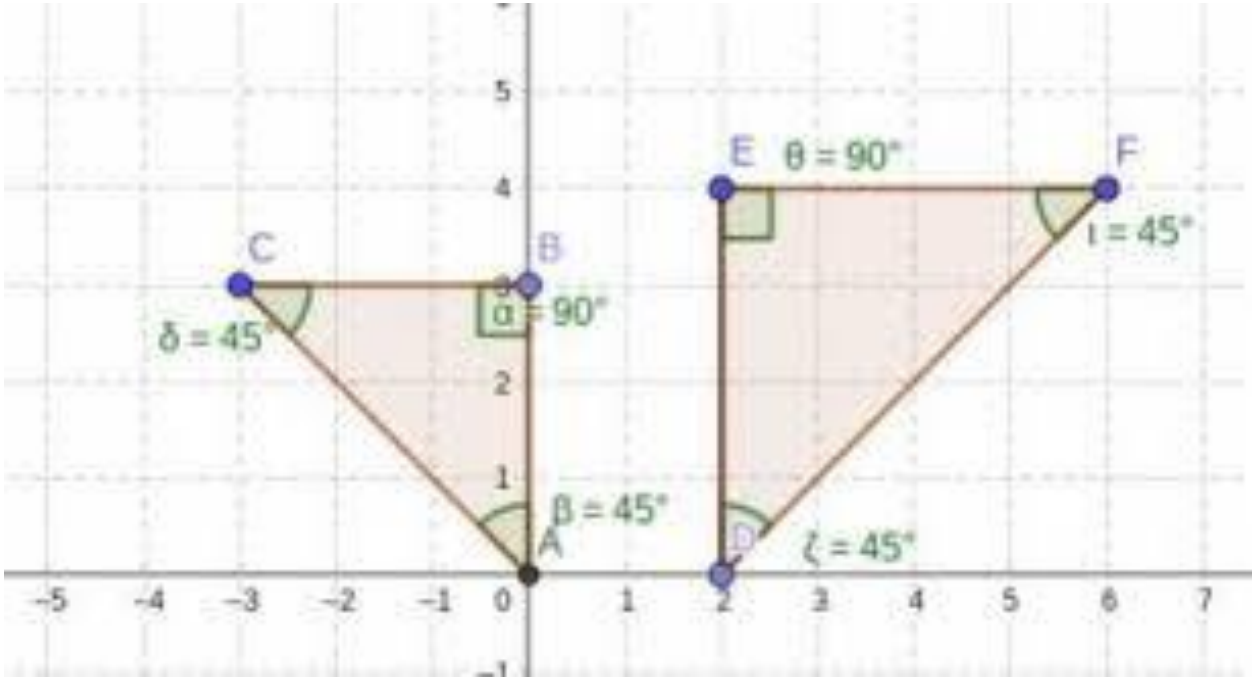
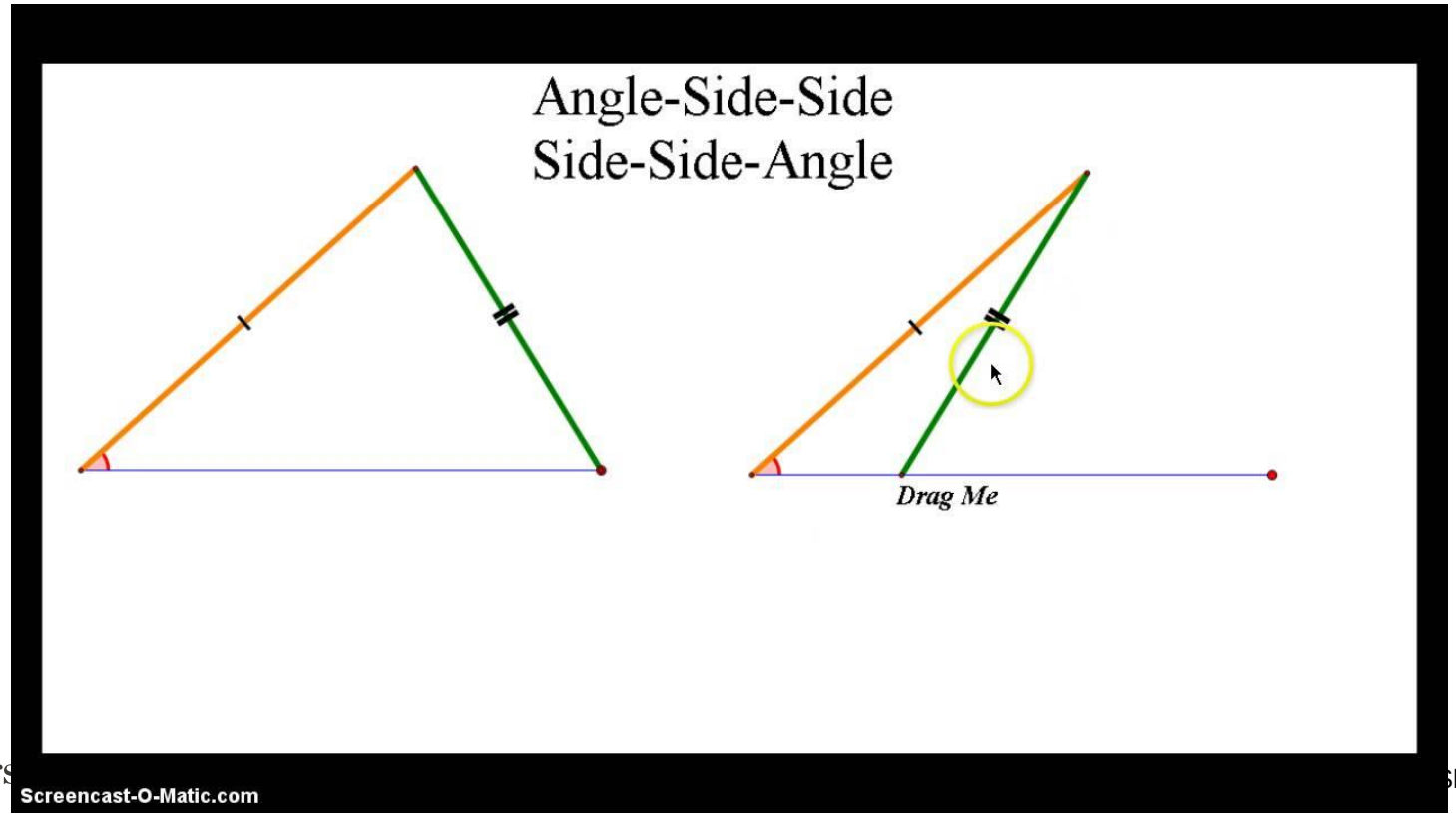
Angle, Angle, Angle (AAA) (Is NOT a congruence property)

- If 3 angles in a triangle are congruent to 3 corresponding angles in another triangle, this is **NOT** enough information to determine congruence. (Instead, it determines similarity.)



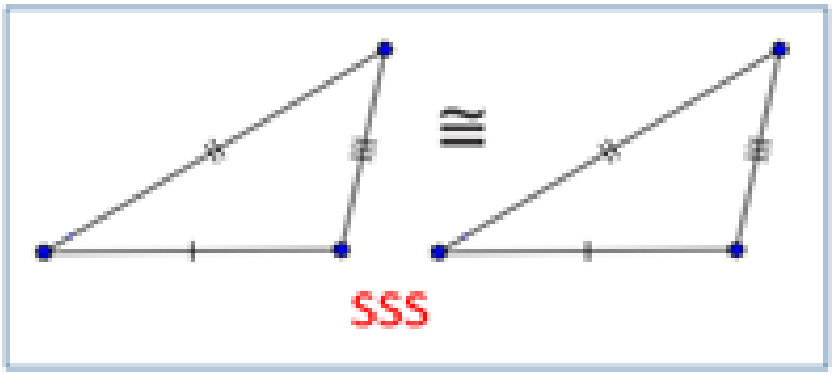
Side, Side, Angle (SSA) Is NOT a Congruence Property

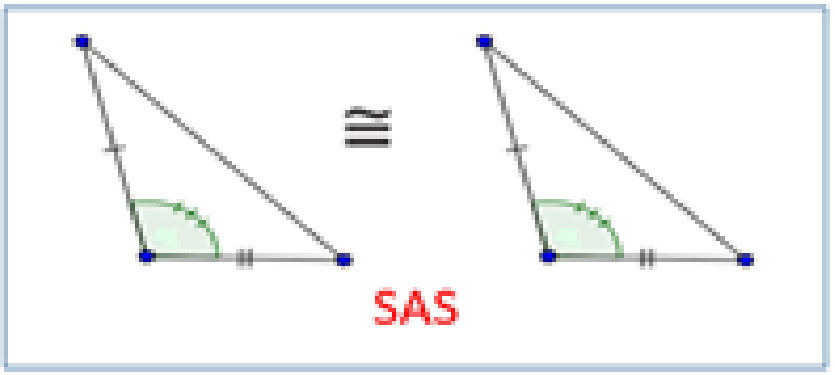
If two sides and the non-included angle of one triangle is congruent to the corresponding two sides and non-included angle of another triangle, the two triangles, this is NOT enough information to determine congruence. In the drawing below, there are two different triangles formed.

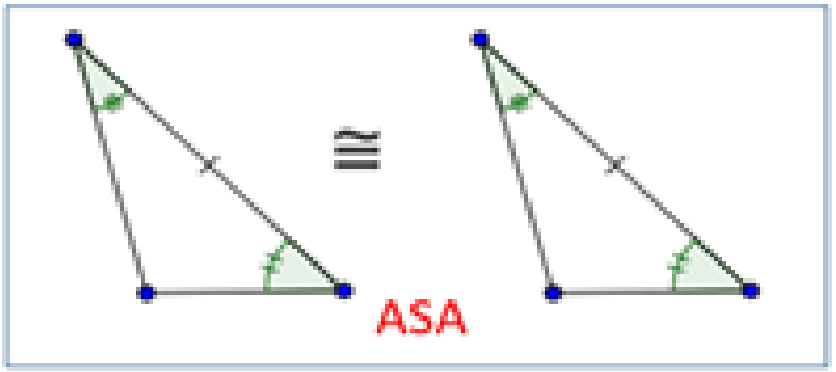


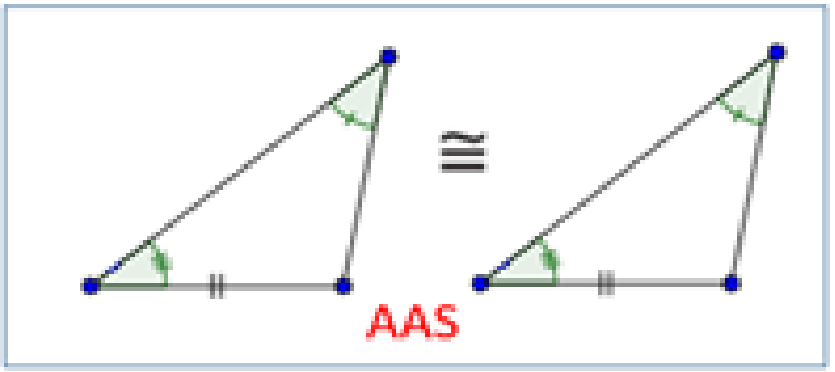
Summary of Congruence Properties

HL Congruence Property is omitted since it is a special case of SSS.









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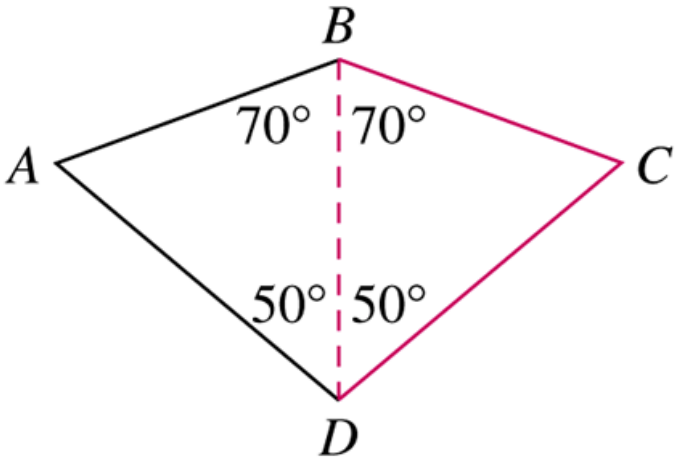
Example 1

Show that the triangles are congruent.



$\angle ABD \cong \angle CBD$, $\overline{BD} \cong \overline{BD}$, and $\angle ADB \cong \angle CDB$.

So, $\Delta ABD \cong \Delta CBD$ by ASA.

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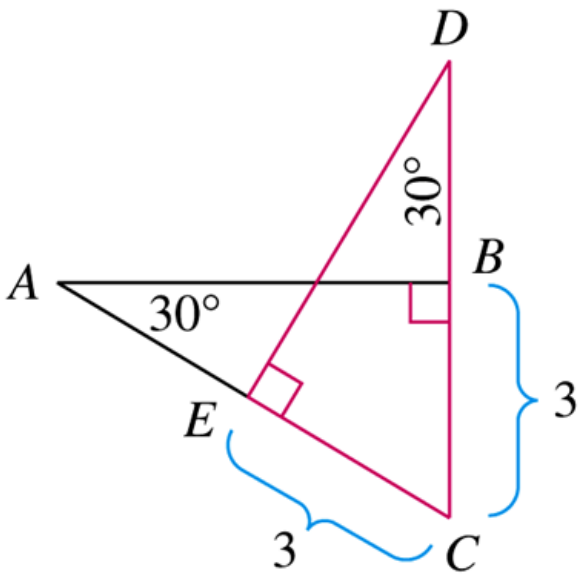
Example 2

Show that the triangles are congruent.



$\triangle ABC$ and $\triangle DEC$

$\angle C \cong \angle C$, $\angle ABC \cong \angle DEC$, and $\overline{EC} \cong \overline{BC}$.

So, $\Delta ABC \cong \Delta DEC$ by AAS.

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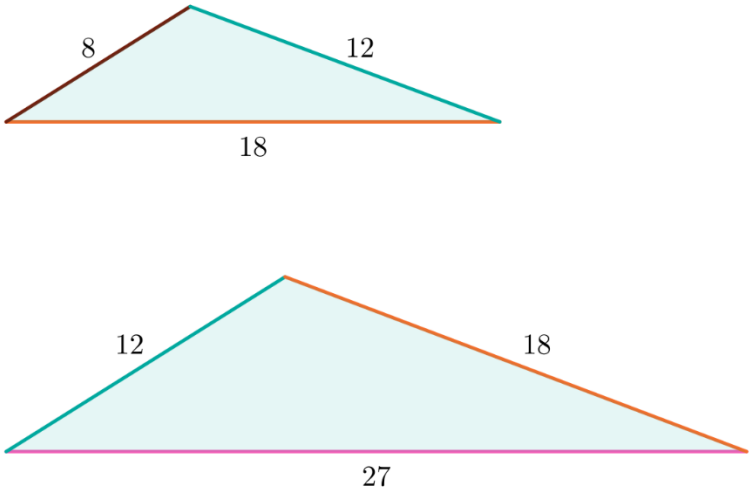
Interesting Fact

It is possible to have 5 parts of one triangle congruent to 5 parts of another triangle without the 6th part being congruent. These are called “5-con” triangles. In the figure below, the corresponding angles are congruent, but note how the congruent sides are not corresponding sides.



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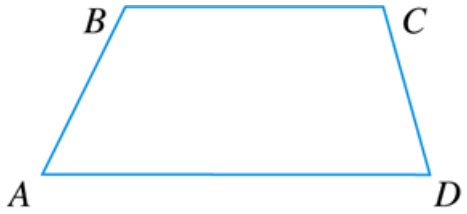
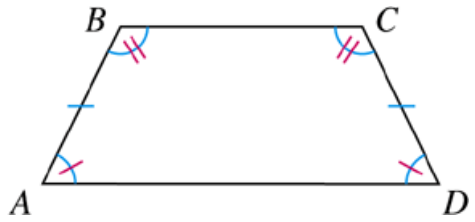
Properties of Quadrilaterals (1 of 6)



Trapezoid: A quadrilateral with at least one pair of parallel sides.

- Consecutive angles between parallel sides are supplementary.



Isosceles trapezoid: A trapezoid with a pair of congruent base angles

- Each pair of base angles are congruent.
- A pair of opposite sides are congruent.
- If a trapezoid has congruent diagonals, it is isosceles.

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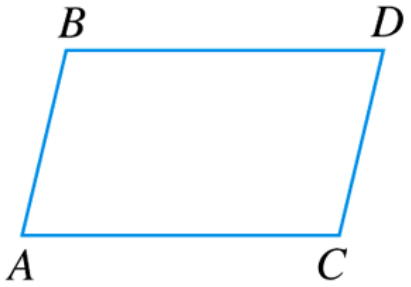
Properties of Quadrilaterals (2 of 6)



Parallelogram: A quadrilateral in which each pair of opposite sides is parallel

- A parallelogram has all the properties of a trapezoid.
- Opposite sides are congruent.
- Opposite angles are congruent.
- Diagonal bisect each other.
- A quadrilateral in which the diagonals bisect each other is a parallelogram.

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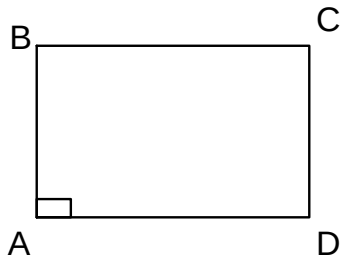
Properties of Quadrilaterals (3 of 6)



Rectangle: A parallelogram with a right angle

- A rectangle has all the properties of a parallelogram.
- All the angles of a rectangle are right angles.
- A quadrilateral in which all the angles are right angles is a rectangle.
- The diagonals of a rectangle are congruent and bisect each other.
- A quadrilateral in which the diagonals are congruent and bisect each other is a rectangle.

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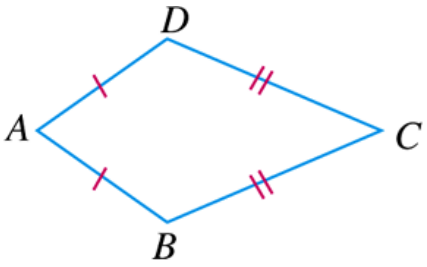
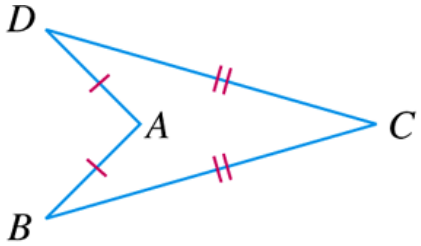
Properties of Quadrilaterals (4 of 6)



convex kite



concave kite

Kite: A quadrilateral with two adjacent sides congruent and the other two sides also congruent.

- Lines containing the diagonals are perpendicular to each other.
- A line containing one diagonal is a bisector of the other diagonal.
- One diagonal bisects nonconsecutive angles.
- A quadrilateral in which the line containing one diagonal is the perpendicular bisector of the other diagonal is a kite.

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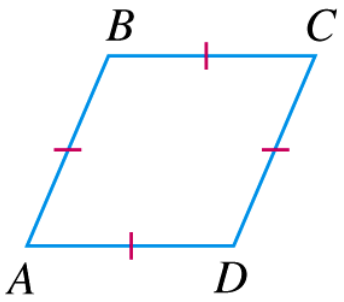
Properties of Quadrilaterals (5 of 6)



Rhombus: A parallelogram with two adjacent sides congruent

- A rhombus has all the properties of a parallelogram and a kite.
- A quadrilateral in which all the sides are congruent is a rhombus.
- The diagonals of a rhombus are perpendicular to and bisect each other. Each diagonal bisects opposite angles.
- A quadrilateral in which the diagonals are perpendicular to and bisect each other is a rhombus.

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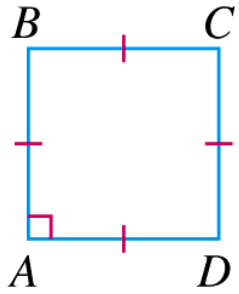
Properties of Quadrilaterals (6 of 6)



Square: A rectangle with all sides congruent

- A square has all the properties of a parallelogram, a rectangle, and a rhombus.
- A rhombus with a right angle is a square.

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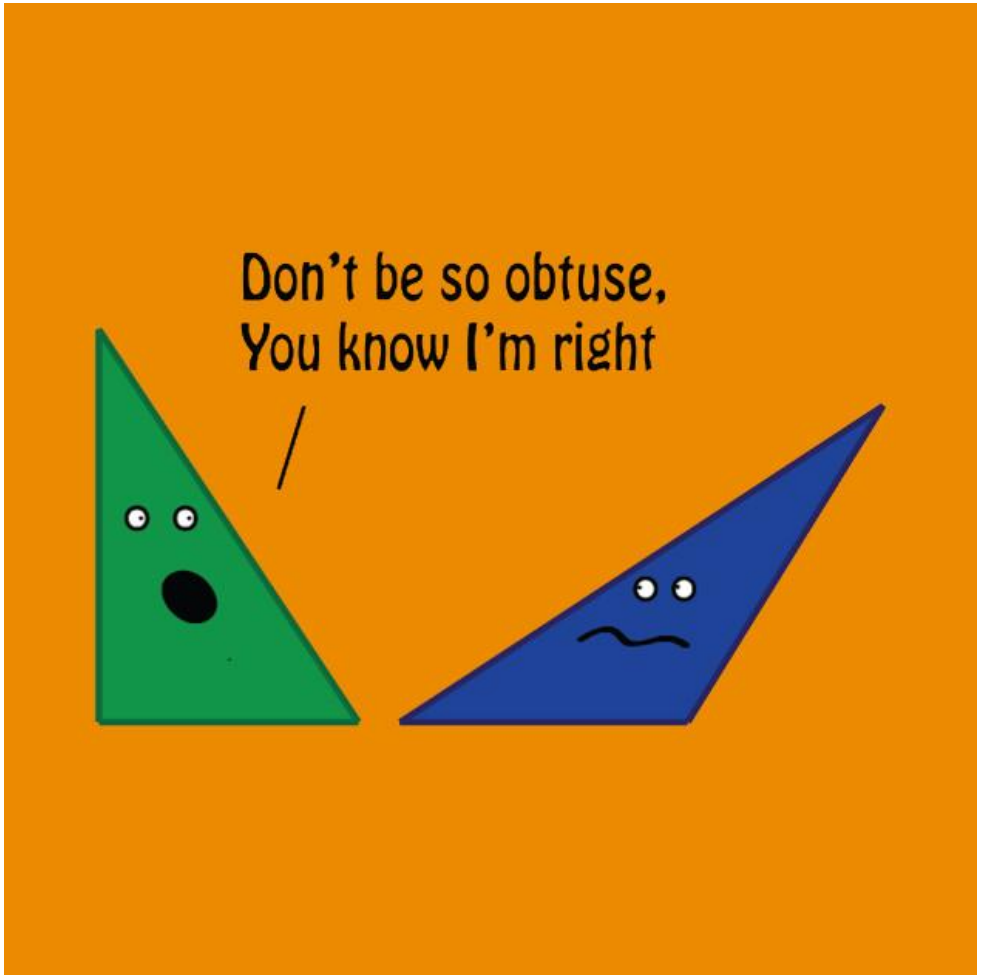
That's it for Section 12-2



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