Section 13-4 Surface Areas

Students will be able to understand and explain

- Surface areas of right prisms, right circular cylinders, right regular pyramids, right circular cones, and spheres.

Surface Area vs Lateral Surface Area

Lateral Surface Area

the sum of the areas of the lateral faces

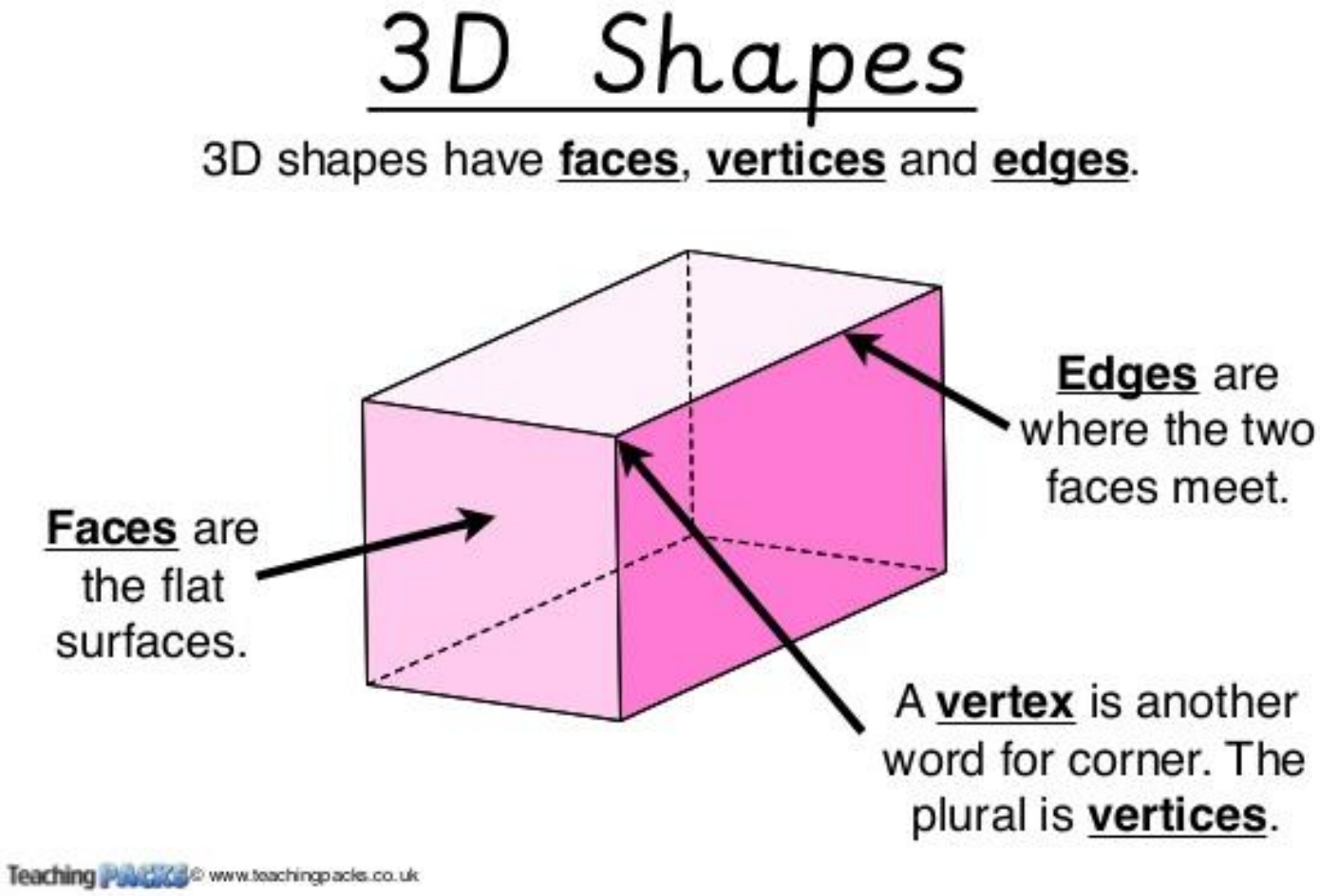
Surface Area

The sum of the lateral surface areas and the area of the bases.

(You can think of surface area as the amount of wrapping paper needed to cover all sides of a gift.)

Area is for 2 dimensional shapes whereas Surface Area is for 3 dimensional shapes.

Faces, Edges, and Vertices

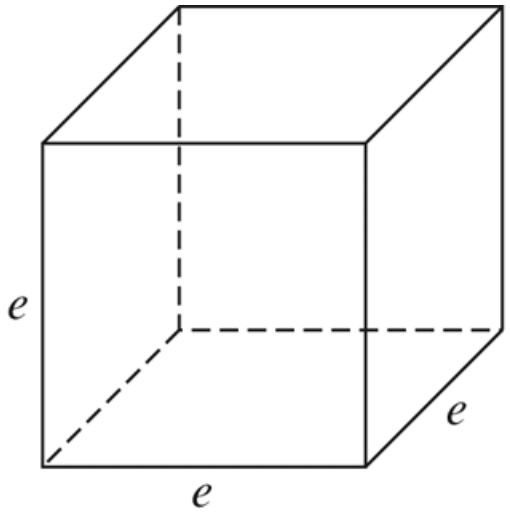


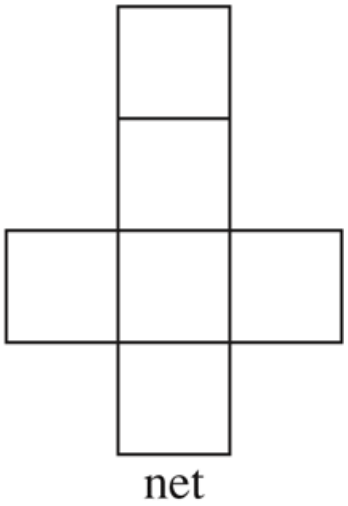
Surface Area of a Cube

Cube

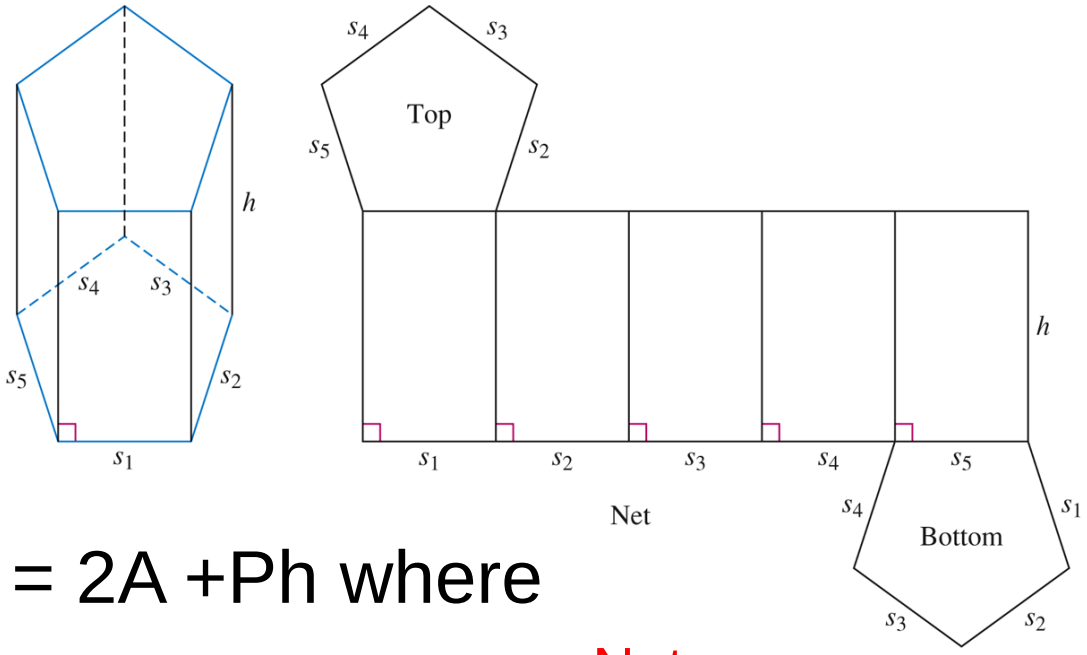
$$\text{Surface area} = 6e^2$$

A cube is made up of 6 squares each with area e^2.





Surface Area of a Prism



Surface Area = 2A +Ph where

P = the perimeter of the base

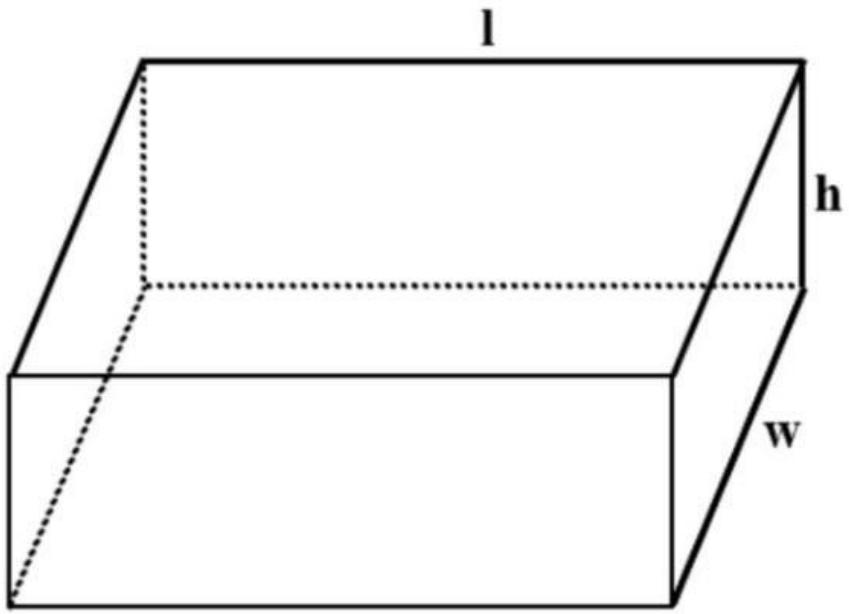
h = the height of the prism

A = the area of the base.

Note:
2A=area of upper & lower bases
Ph=area of the big rectangle made by combining all the lateral sides

Rectangular Prism

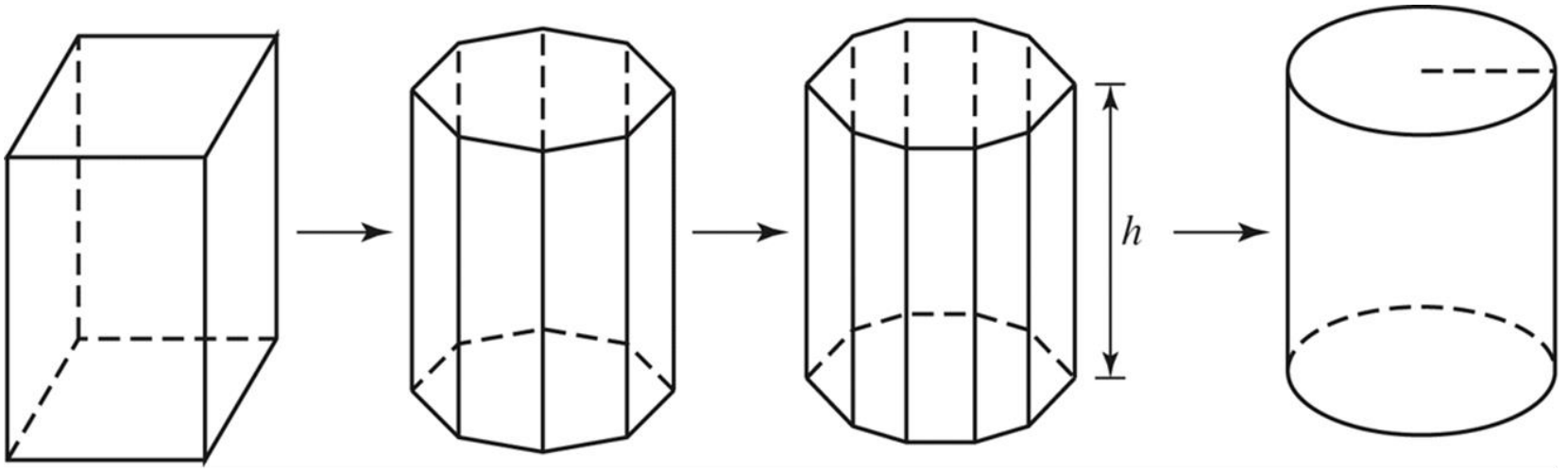
Surface Area of a Rectangular Prism



$$SA = 2lw + 2wh + 2lh$$

This is the same as the 2A+Ph formula. 2A = 2ℓw & P=2ℓ+2w

Surface Area of a Cylinder



Right regular prisms

Right circular cylinder

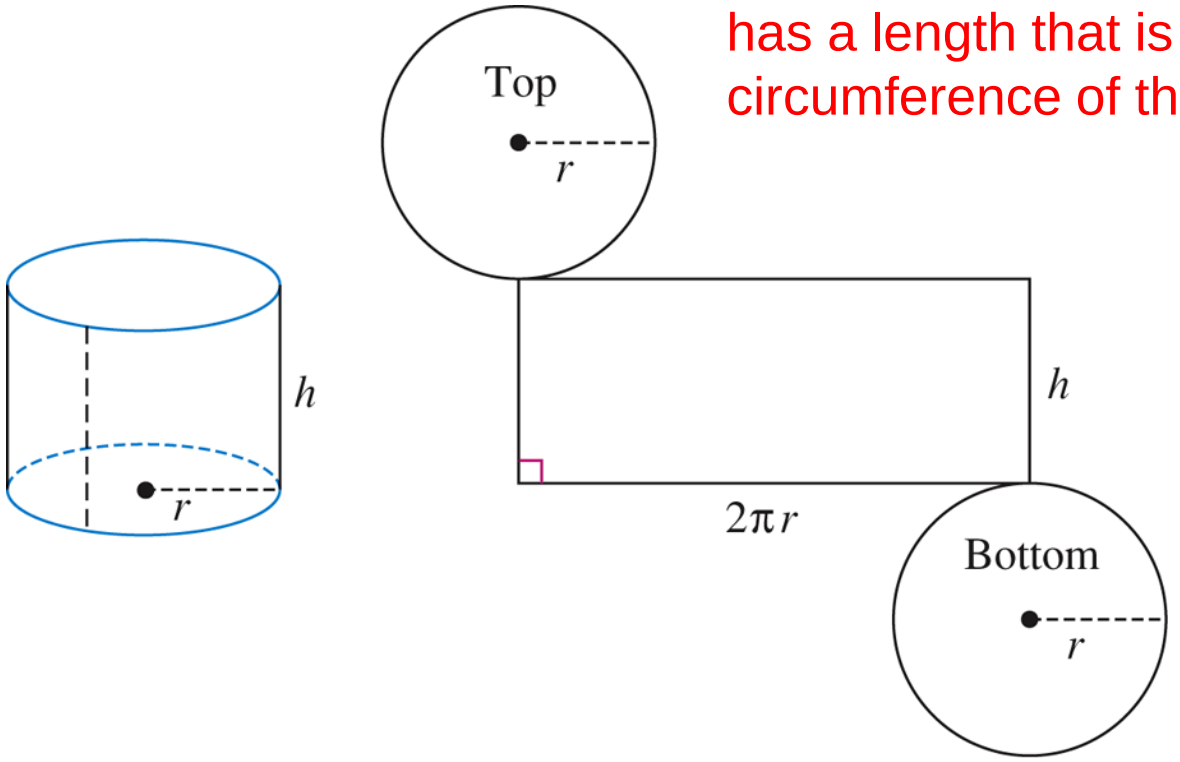
Surface Area of a Right Circular Cylinder

Right circular cylinder

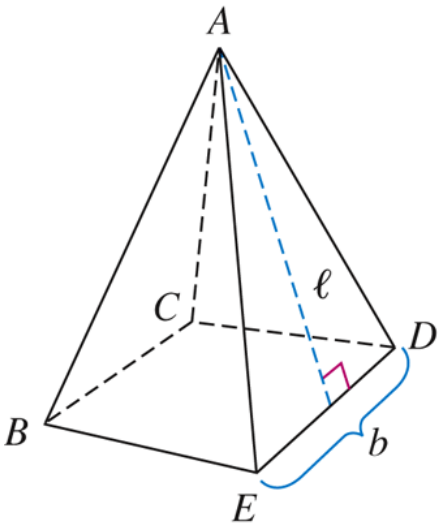
$$\text{Surface area} = 2\pi r^2 + 2\pi rh$$

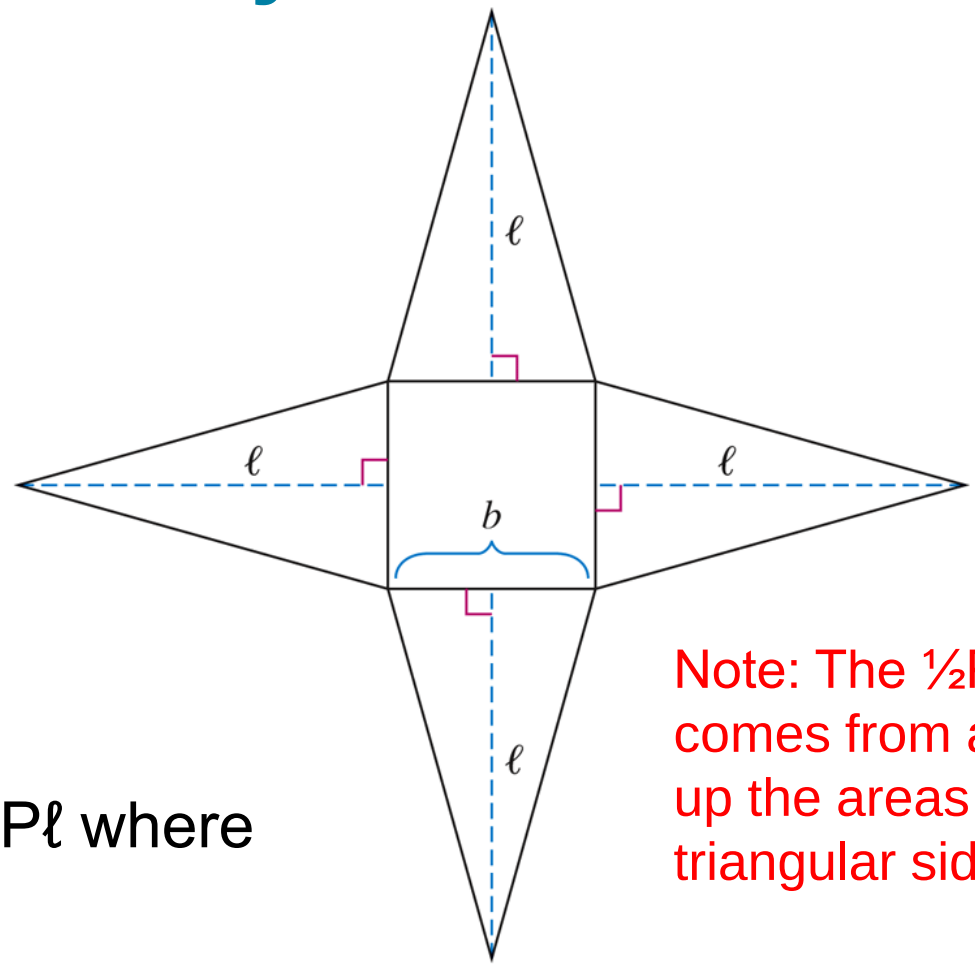
$2\pi r^2$=area of upper and lower circular bases

$2\pi rh$ = area of the rectangular lateral side. This rectangle has a length that is the circumference of the base.



Surface Area of a Pyramid

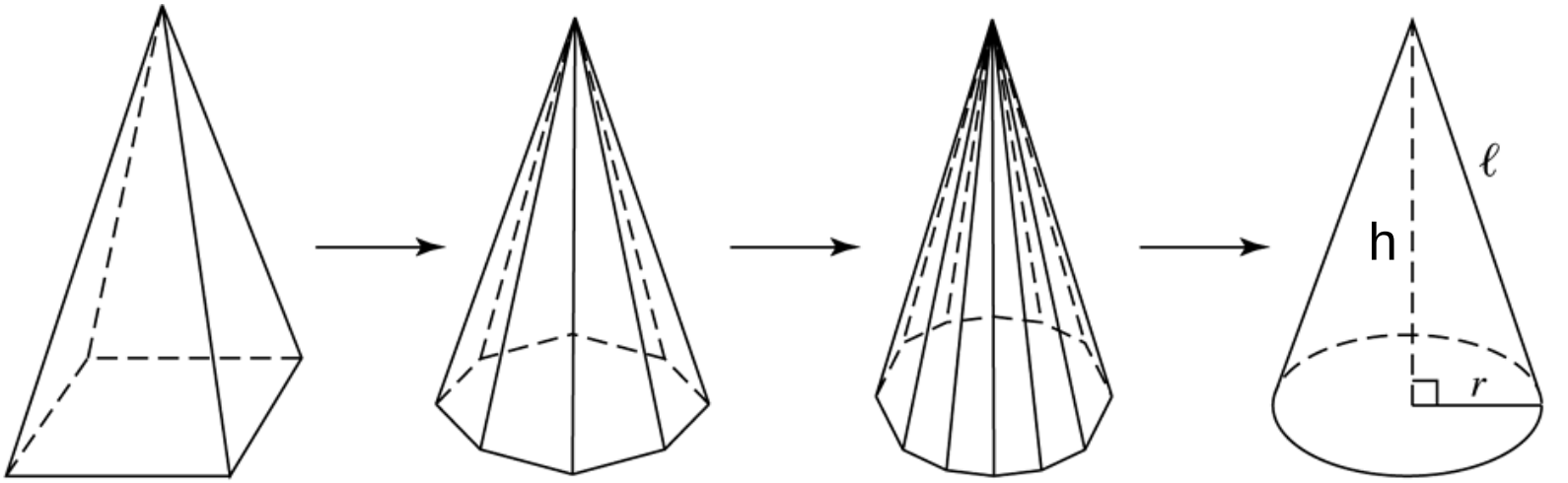




Note: The ½Pℓ comes from adding up the areas of the triangular sides

Surface Area = A + ½ Pℓ where
A = Area of the base
P = Perimeter of the base
ℓ = slant height of a triangular lateral face

Surface Area of a Cone (1 of 2)



Right regular pyramids

Right circular cone

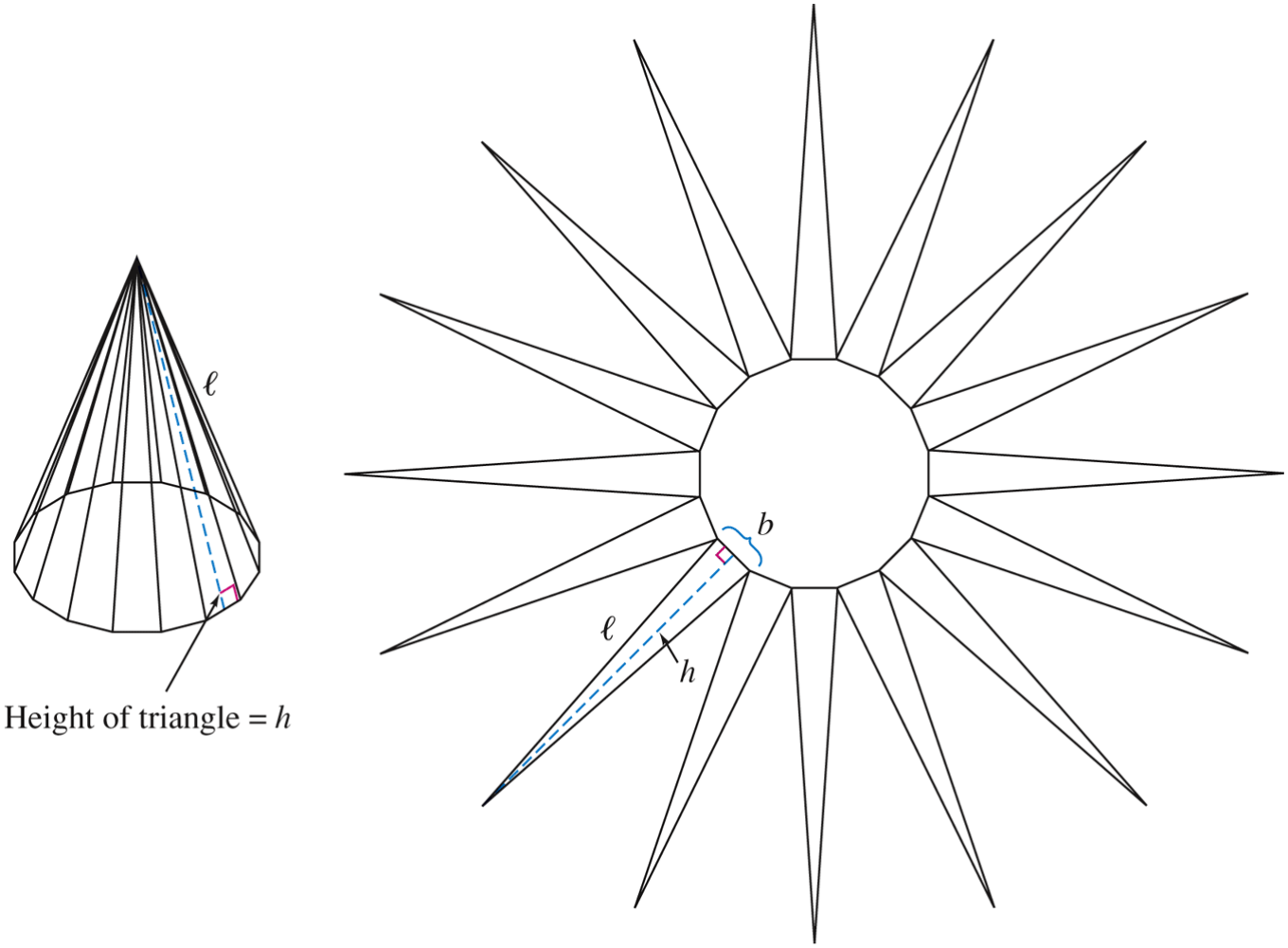
Surface Area = A + ½ Cℓ
where
A = area of the base
C = circumference of the base
ℓ = slant height

This formula can also be written as:

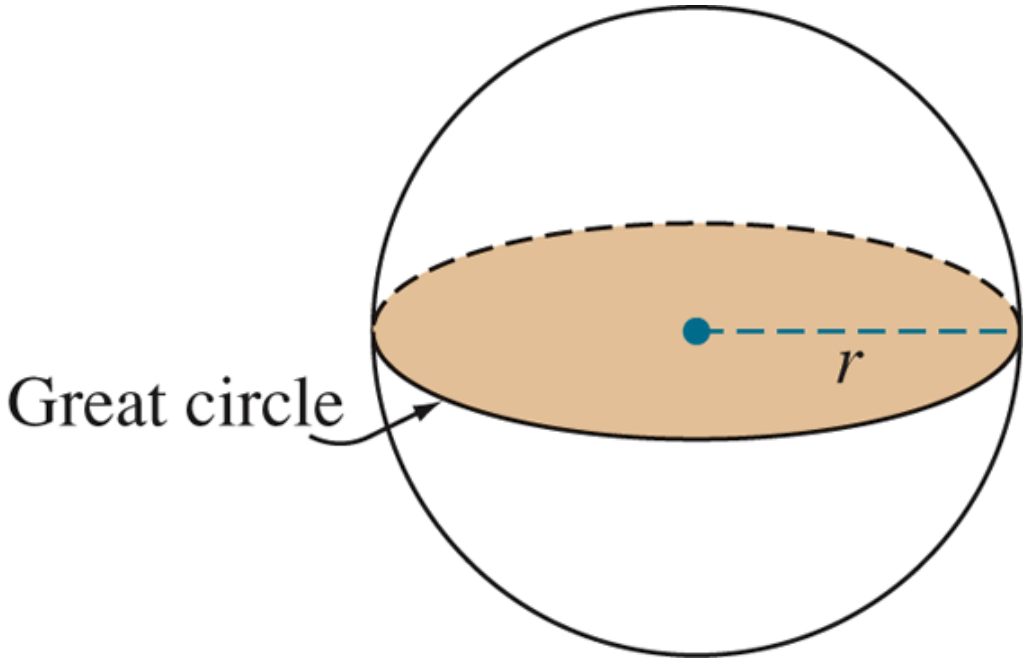
$SA = \pi r^2 + \pi r\sqrt{h^2 + r^2}$

or $SA = \pi r^2 + \pi r\,\ell$

Surface Area of a Cone (2 of 2)



Surface Area of a Sphere



$$\text{Surface area} = 4\pi r^2$$

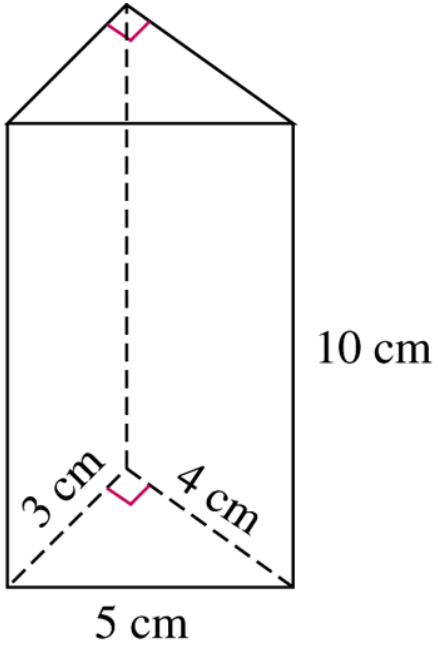
Example 1

Find the surface area of the prism.

$A = \frac{1}{2}(3 \cdot 4) = 6\text{cm}^2$

$P = 3 + 4 + 5 = 12\text{cm}$

h = 10 cm

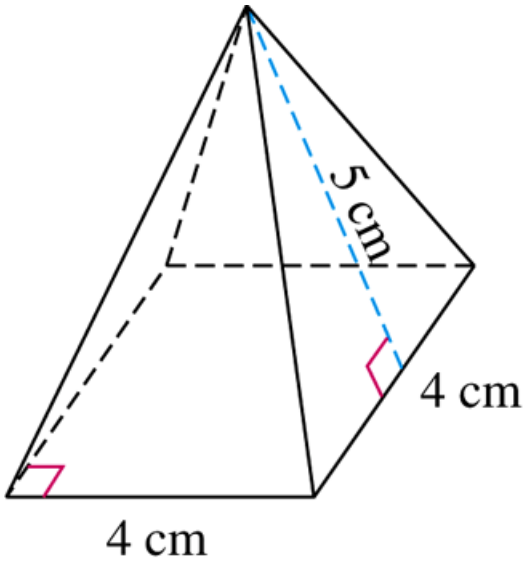


Right triangular prism

$SA = 2A + Ph = 2(6) + 12(10) = 132\text{cm}^2$

Example 2

Find the surface area of the pyramid.



Right square pyramid

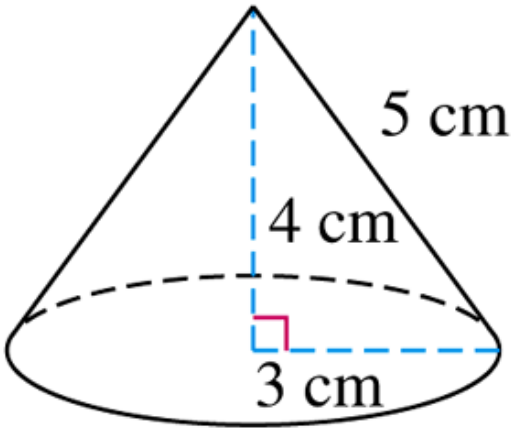
$$A = 4 \cdot 4 = 16\text{cm}^2$$
$$P = 4 \cdot 4 = 16\text{cm}$$
$$\ell = 5\ cm$$

$$SA = \frac{1}{2}P\ell + A = \frac{1}{2}(16)(5) + 16 = 56\text{cm}^2$$

Example 3

Find the surface area of the cone.

r = 3cm
ℓ = 5 cm

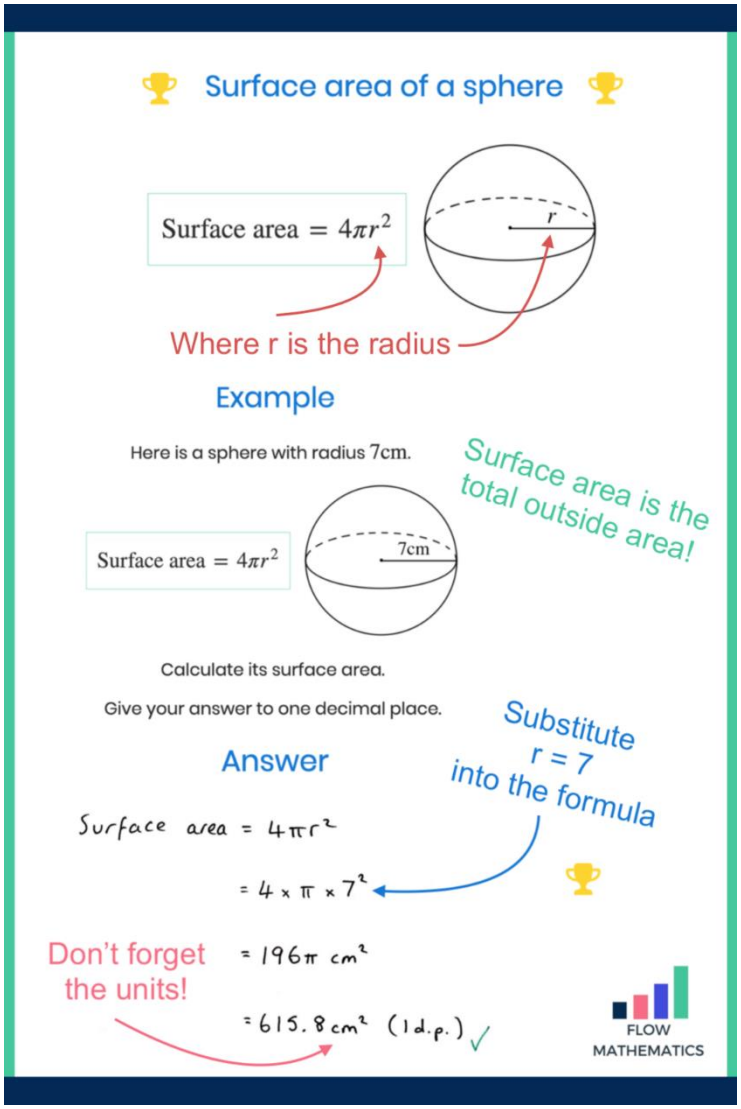


Right circular cone

$$SA = \pi r^2 + \pi rl$$

$$SA = \pi(3)^2 + \pi(3)(5) = 24\pi \text{ cm}^2$$

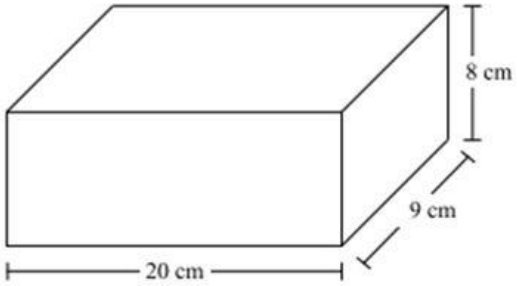
Extra Examples I found… (1 of 3)



Extra Examples I found… (2 of 3)

Practice Problem 1

- Find the surface area of the rectangular prism shown to the right.



$$S.A. = 2lw + 2lh + 2hw$$
$$S.A. = 2(20)(9) + 2(20)(8) + 2(8)(9)$$
$$S.A. = 360 + 320 + 144$$
$$S.A. = 824cm^2$$

Extra Examples I found… (3 of 3)

Example 1: Finding Surface Area of a Pyramid

- Find the surface area of a square pyramid with base edges 5 m and slant height 3 m.

$$S = L + B$$

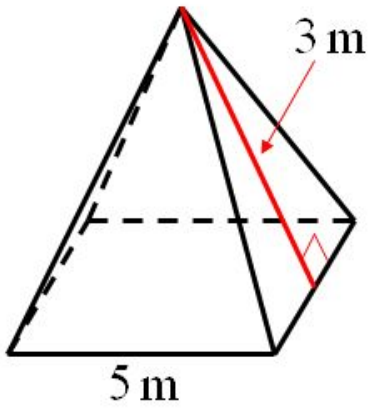
$$L = \frac{1}{2} P\ell \qquad B = s^2$$

$$P = 4(5) = 20 \qquad B = 5^2$$

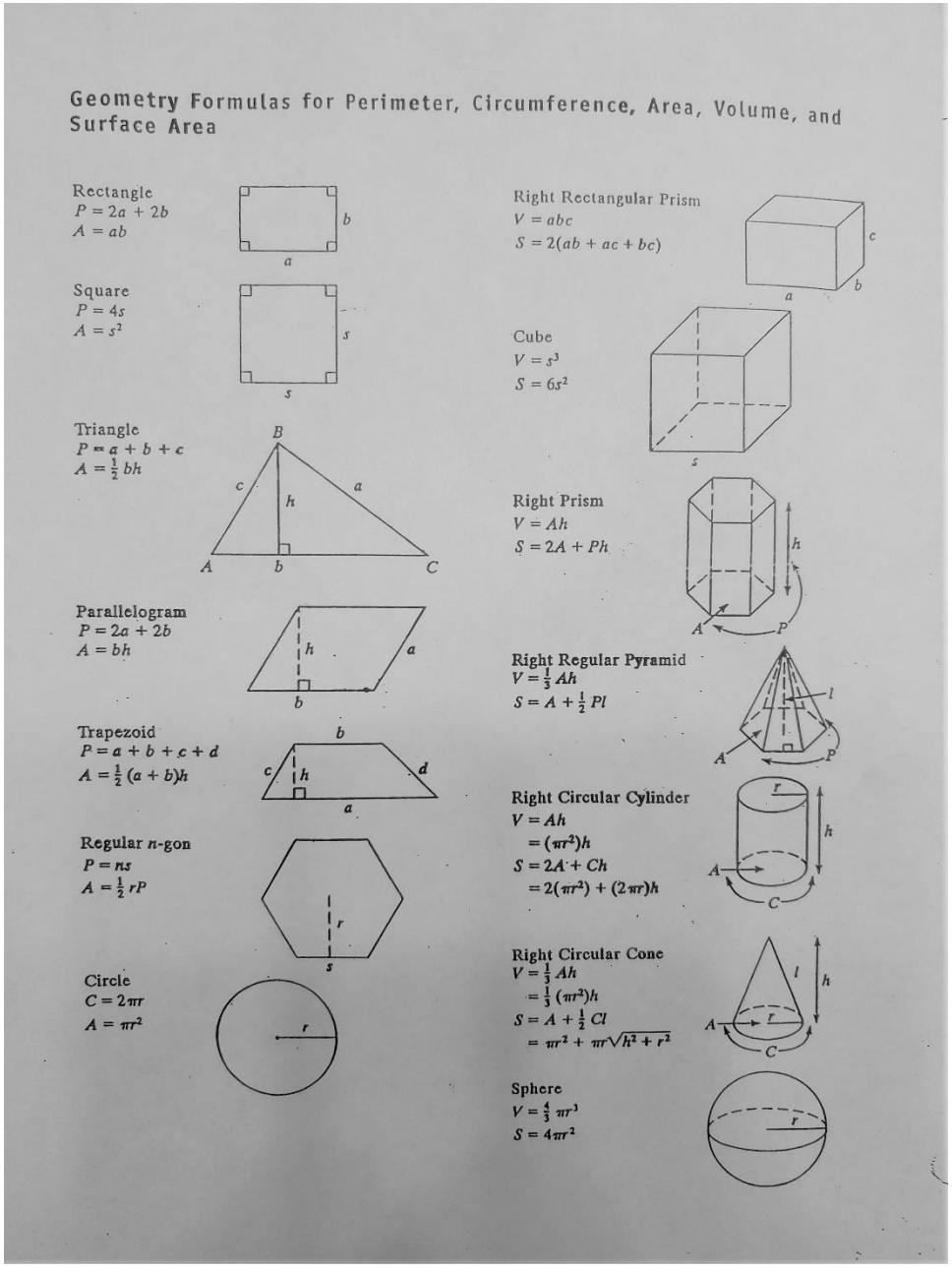
$$L = \frac{1}{2} 20(3) = 30\,\text{m}^2 \qquad B = 25\,\text{m}^2$$

$$S = 30 + 25$$

$$\boxed{S = 55\,\text{m}^2}$$



This ends Section 13-4



Geometry Formulas for Perimeter, Circumference, Area, Volume, and Surface Area

Rectangle
$P = 2a + 2b$
$A = ab$

Square
$P = 4s$
$A = s^2$

Triangle
$P = a + b + c$
$A = \frac{1}{2}bh$

Parallelogram
$P = 2a + 2b$
$A = bh$

Trapezoid
$P = a + b + c + d$
$A = \frac{1}{2}(a + b)h$

Regular n-gon
$P = ns$
$A = \frac{1}{2}rP$

Circle
$C = 2\pi r$
$A = \pi r^2$

Right Rectangular Prism
$V = abc$
$S = 2(ab + ac + bc)$

Cube
$V = s^3$
$S = 6s^2$

Right Prism
$V = Ah$
$S = 2A + Ph$

Right Regular Pyramid
$V = \frac{1}{3}Ah$
$S = A + \frac{1}{2}Pl$

Right Circular Cylinder
$V = Ah$
$= (\pi r^2)h$
$S = 2A + Ch$
$= 2(\pi r^2) + (2\pi r)h$

Right Circular Cone
$V = \frac{1}{3}Ah$
$= \frac{1}{3}(\pi r^2)h$
$S = A + \frac{1}{2}Cl$
$= \pi r^2 + \pi r\sqrt{h^2 + r^2}$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$