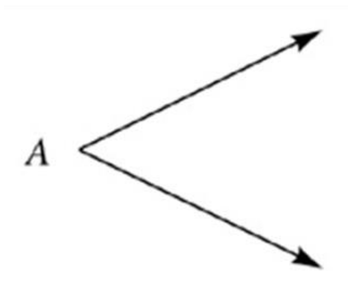
Section 12-3 Additional Constructions

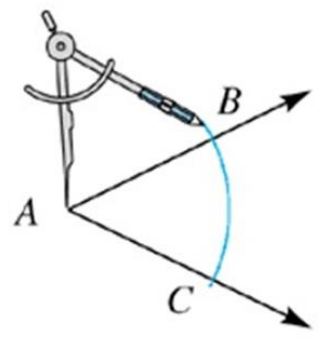
Students will be able to understand and explain

- Constructing angle bisectors, parallel lines, and perpendicular lines.
- Angle bisector properties.

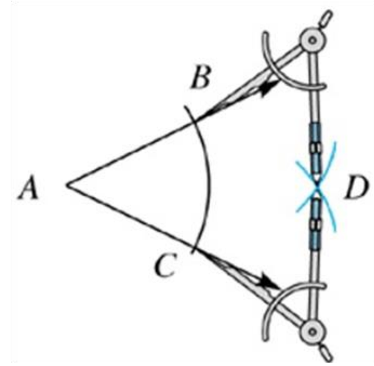
Constructing Angle Bisectors



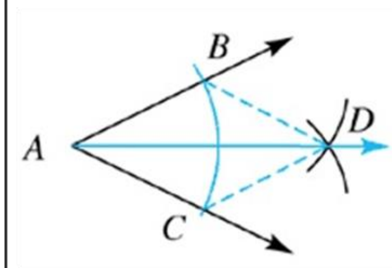
To bisect a given angle, we make $\angle A$ an angle of a rhombus.



With the pointer at *A*, draw any arc intersecting the angle at *B* and *C*, giving three vertices of the desired rhombus.



Draw an arc with center at *B* and radius AB. Then, draw an arc with center at *C* and radius AB. The arcs intersect at *D*, the fourth vertex of the rhombus.

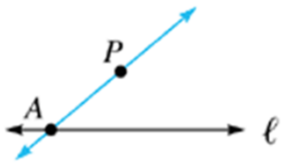
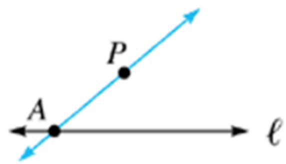
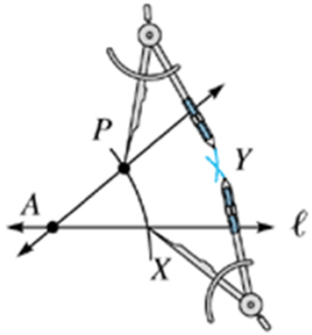
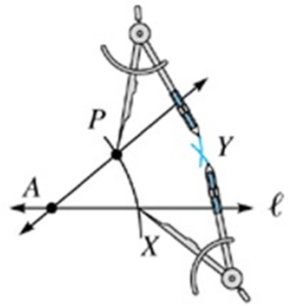
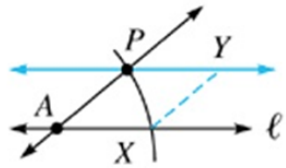


Connect *A* with D. $\overrightarrow{AD}$ is the angle bisector of $\angle A$ in rhombus $ABDC$.

Note: If you connect points B and D the line segment formed is parallel to line segment AC.

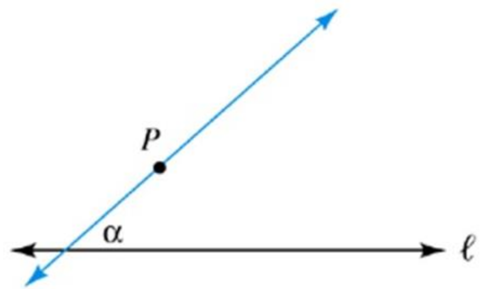
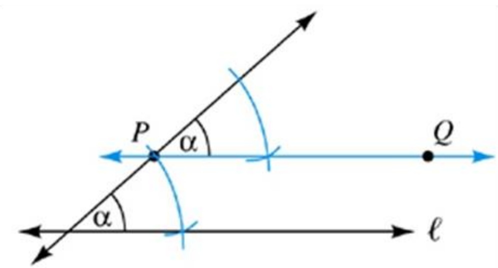
Constructing Parallel Lines (1 of 2)

Rhombus Method (also sometimes called the angle bisector method)

				
To construct a line through *P* parallel to ℓ, make *P* a vertex of a rhombus so that one of its sides is on ℓ.	Through *P*, draw any line that intersects ℓ in *A*. $\overline{PA}$ will be a side of the rhombus.	Draw an arc with the pointer at *A* and with radius $\overline{AP}$ to mark the third vertex, *X*, of the desired rhombus.	With the same opening of the compass, draw intersecting arcs, first with the pointer at *P* and then with the pointer at *X* to find *Y*, the fourth vertex of the rhombus.	Draw $\overleftrightarrow{PY}$. $\overleftrightarrow{PY} \parallel \ell$ in rhombus *APYX*.

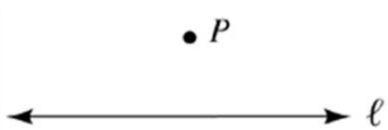
Constructing Parallel Lines (2 of 2)

Corresponding angle method—this technique uses the congruent angle construction from 12-1.

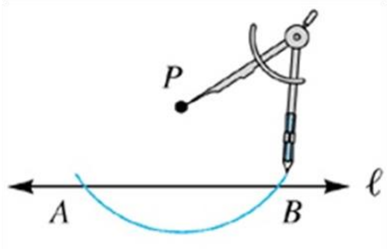
		
To construct a line through P parallel to ℓ:	Through P, draw a line that intersects ℓ.	Copy angle α at point P to form corresponding angles. $\overleftrightarrow{PQ} \parallel \ell$.

Constructing Perpendicular Lines

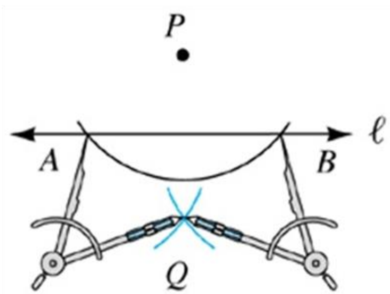
Constructing a perpendicular to a line from a point **NOT** on the line



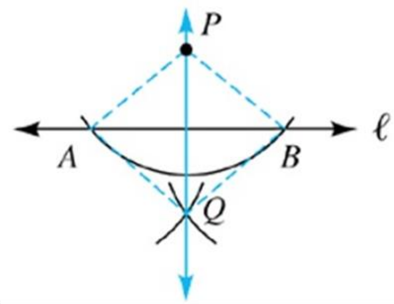
To construct a perpendicular to ℓ through *P*, we make *P* a vertex of a rhombus.



Draw an arc with center at *P* that intersects the line at two points, *A* and *B*, two more vertices of the rhombus.



With the same compass opening, make two intersecting arcs, one with center at *A* and the other with center at *B*. The arcs intersect at Q, the final vertex of rhombus *PAQB*.

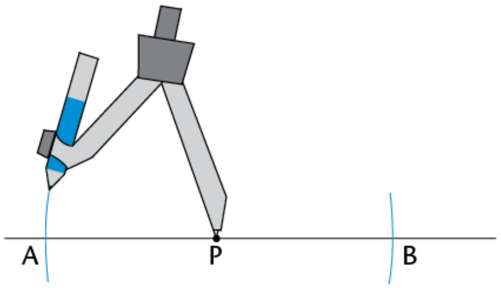


Connect *P* with Q. $\overleftrightarrow{PQ}$ is the required line. $\overleftrightarrow{PQ}$ And $\overline{AB}$ are perpendicular diagonals in rhombus *PAQB*.

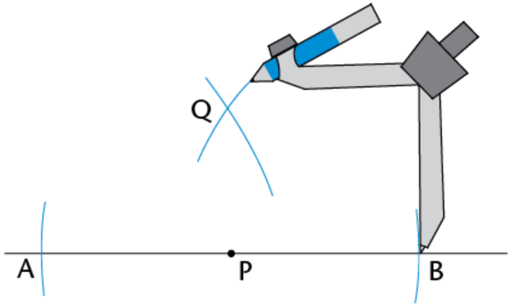
Constructing Perpendicular Lines

Constructing a perpendicular to a line from a point on the line

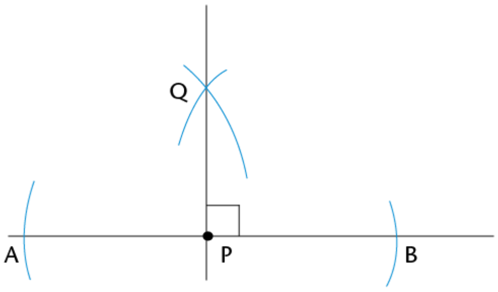
Place your compass on the given point (P). Draw an arc across the line on each side of the given point. Do not adjust the compass width when drawing the second arc.



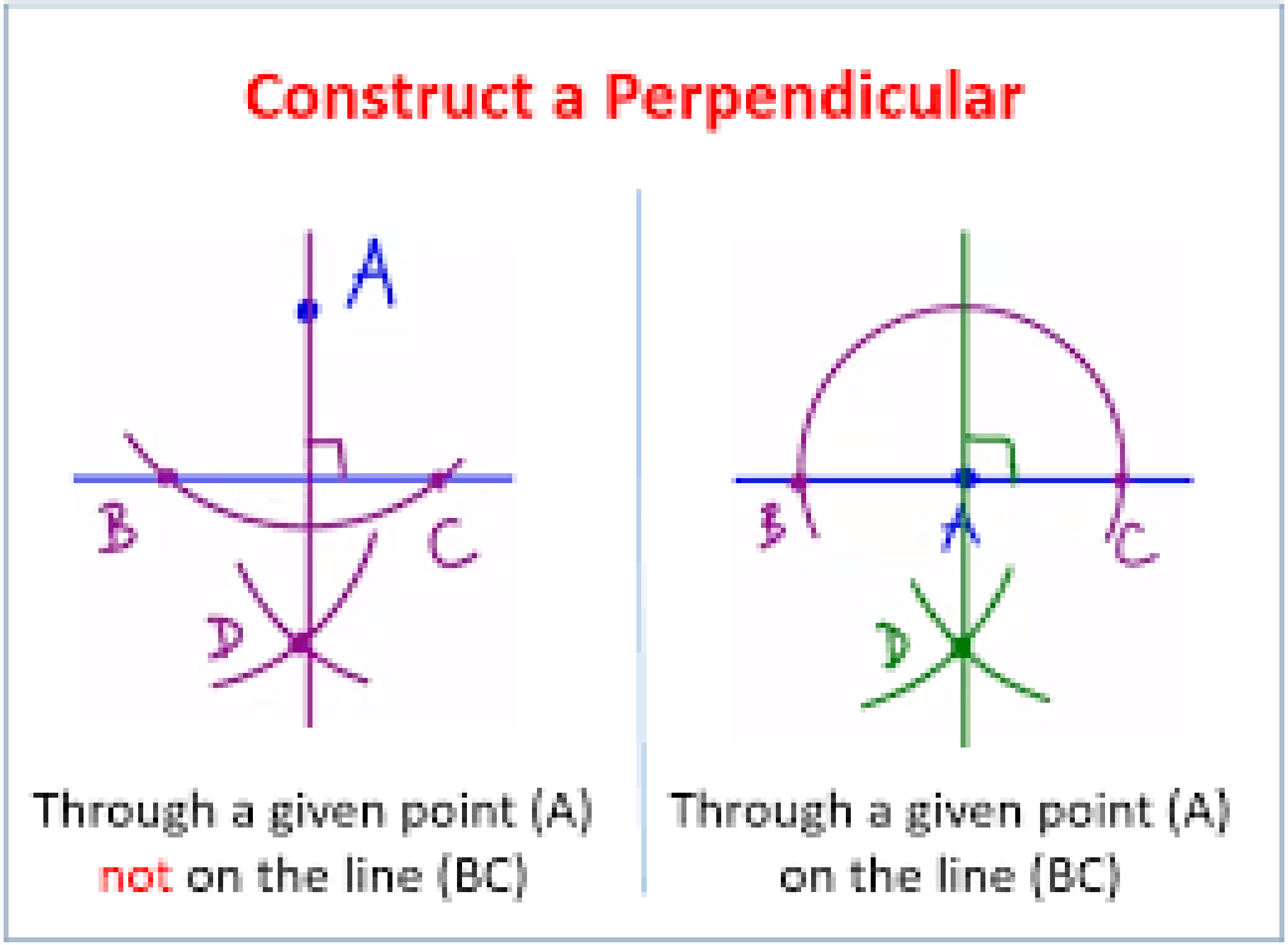
Open your compass so that it is wider than the distance from one of the arcs to the point P. Place the compass on each arc and draw an arc above or below the point P. The two new arcs will intersect.



Use your ruler to join the given point (P) and the point where the arcs intersect (Q).



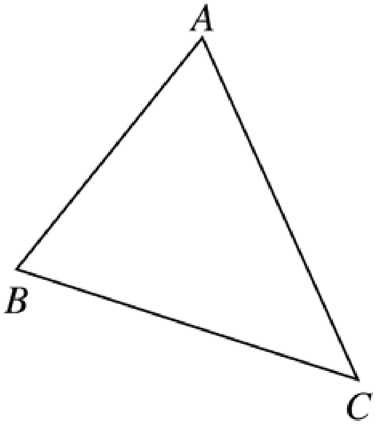
Constructing a Perpendicular: Summary

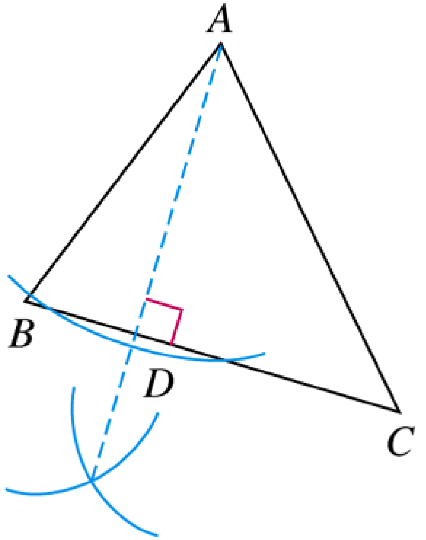


Example 1

- Given triangle *ABC*, construct an altitude from vertex *A*.

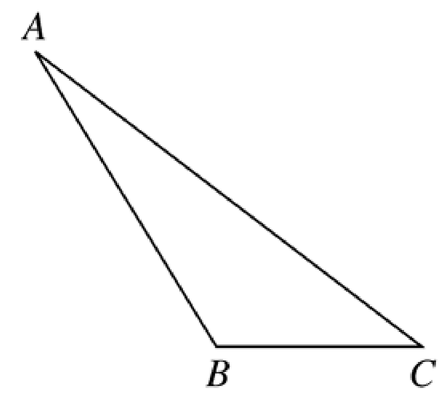
- An altitude is the segment perpendicular from a vertex to the line containing the opposite side of a triangle, so construct a perpendicular from point *A* to the line containing $\overline{BC}$.



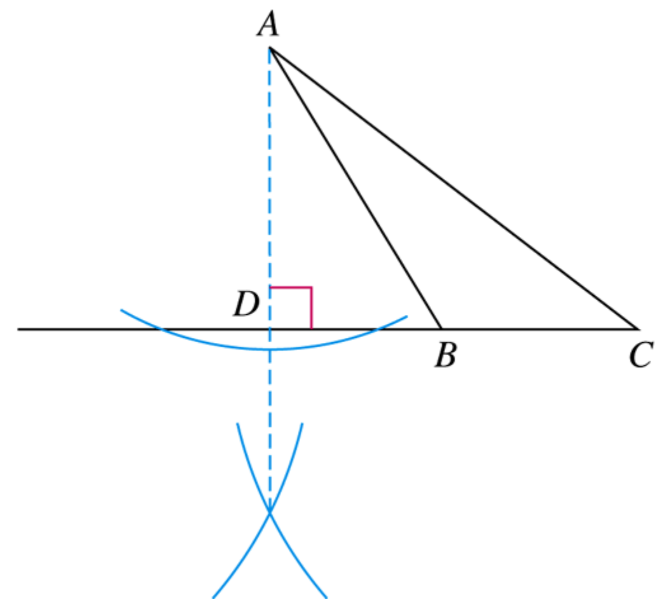


Example 2

Given triangle *ABC*, construct an altitude from vertex *A*.

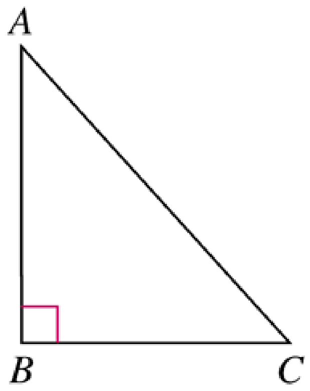


Notice that the altitude $\overline{AD}$ does not intersect the interior of triangle *ABC*.



Example 3

Given triangle *ABC*, construct an altitude from vertex *A*.

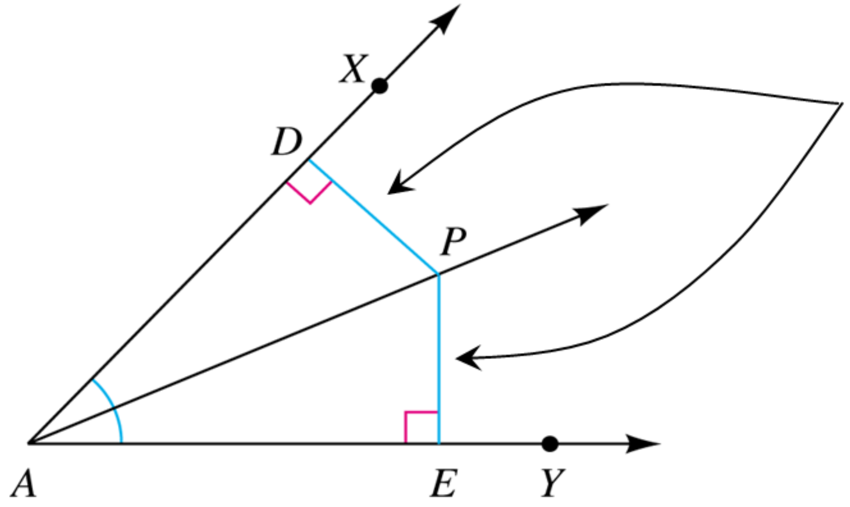


Triangle *ABC* is a right triangle. The altitude from vertex *A* is the side $\overline{AB}$.

Properties of Angle Bisectors

ΔADP is congruent to Δ AEP by SAA.
(Shared side AP, bisected angle A, right angle)

Any point *P* on an angle bisector is equidistant from the sides of the angle and vice versa.



These distances are the same.

Links to Video Constructions (1 of 2)

- Angle Bisector (1:22)
  https://www.youtube.com/watch?v=qBw0Ly-wF4U
- Parallel lines (corresponding angle method) (1:26)
  https://www.youtube.com/watch?v=b7iFyR87FTQ
- Parallel lines (rhombus method) (1:30)
  https://www.youtube.com/watch?v=kJCg63d1fqE
- Perpendicular from a point NOT on the given line (4:02)
  https://www.youtube.com/watch?v=Rr9slPc6dNQ

Links to Video Construction (2 of 2)

- Perpendicular through a point on the given line (1:19) https://www.youtube.com/watch?v=mL34kb-BplE&list=PLC026D398EB6FAC31&index=4
- Altitude of an acute triangle (2:23) https://www.youtube.com/watch?v=JeraWKqZU7A
- Altitude of an obtuse triangle (1:14) https://www.youtube.com/watch?v=zd1rY59WzXI

That ends section 12-3