Section 12-1 Congruence Through Constructions

Students will be able to understand and explain

- Geometric construction and its connections to the SSS, SAS, and HL congruence properties.
- Isosceles triangle properties.
- Altitudes and perpendicular bisectors.
- Circles circumscribing triangles.

Congruence and Similarity

Similar objects (~) have the same shape but not necessarily the same size. (Ex: dime & quarter)

Congruent objects (≅) have the same shape and the same size. (Ex: can of soup & can of corn)

Congruent objects are similar, but similar objects are not necessarily congruent.

Definition (1 of 2)

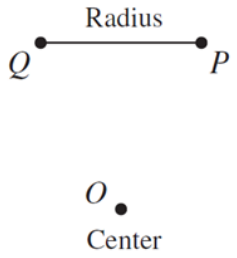
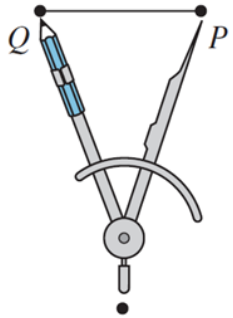
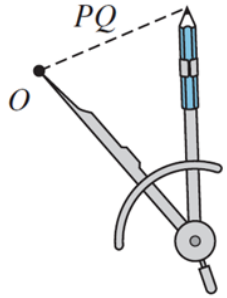
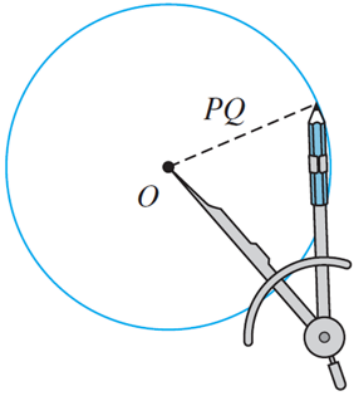
Congruent Segments and Angles

$\overline{AB} \cong \overline{CD}$ if, and only if, the 2 line segments have the same length

$\angle ABC \cong \angle DEF$ if, and only if, the measure of angle ABC is equal to the measure of angle DEF written $m\angle ABC = m\angle DEF$.

Geometric Constructions

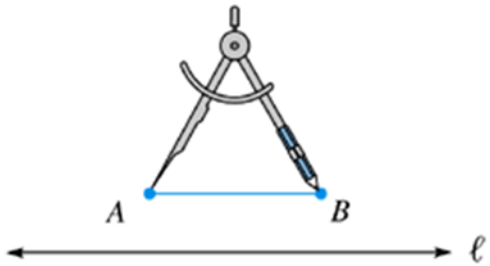
Constructing a circle given its center and radius

			
To construct a circle with center O and radius PQ:	Set the legs of the compass on P and Q to obtain PQ.	Set the pointer at O without changing the opening of the compass.	Hold the pointer at O and move the pencil to draw the circle.

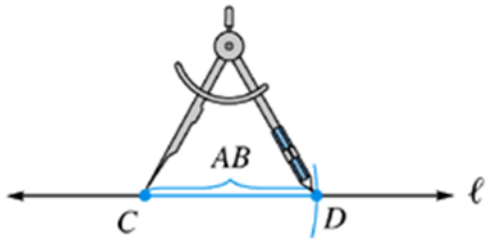
Constructing Segments

There are many ways to draw a segment congruent to another segment – using a ruler, tracing the segment, and the method shown below using a straightedge and compass.

To copy $\overline{AB}$ on any line ℓ:



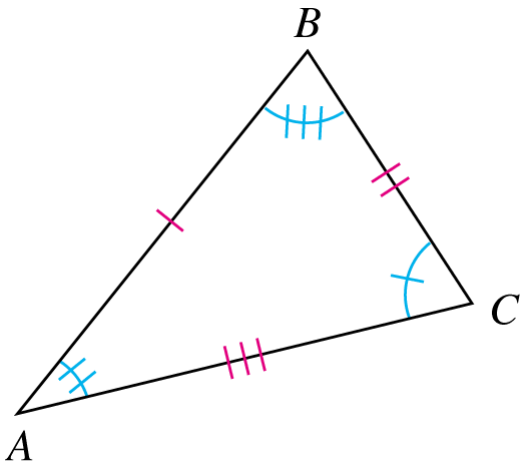
To copy $\overline{AB}$, fix the compass so that the pointer is on A and the pencil is on B (or vice versa).

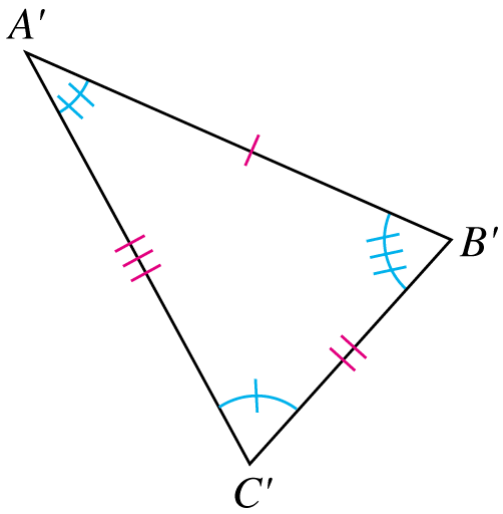


Place the point of the compass at any point C on ℓ and strike an arc to locate point D. $\overline{CD} \cong \overline{AB}$.

Triangle Congruence

Two figures are congruent if it is possible to fit one figure onto the other so that matching parts coincide.





$\Delta ABC \cong \Delta A'B'C'$

Definition (2 of 2)

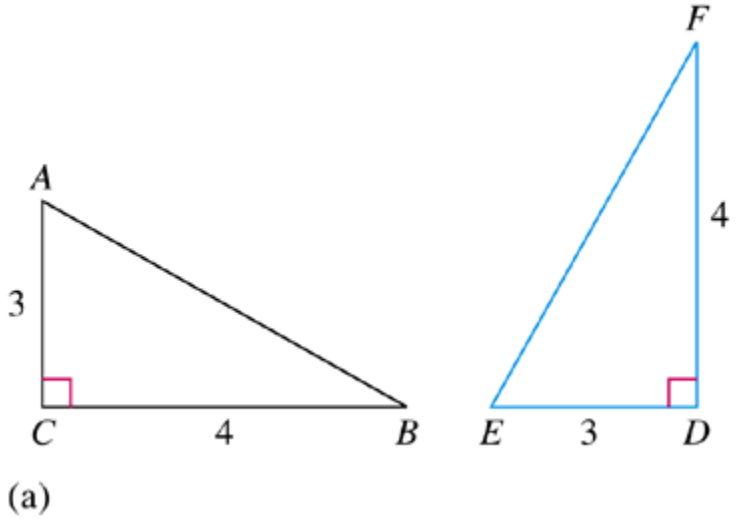
Congruent Triangles

ΔABC is congruent to $\Delta A'B'C'$, written $\Delta ABC \cong \Delta A'B'C'$, if, and only if, $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, $\angle C \cong \angle C'$, $\overline{AB} \cong \overline{A'B'}$, $\overline{BC} \cong \overline{B'C'}$, and $\overline{AC} \cong \overline{A'C'}$.

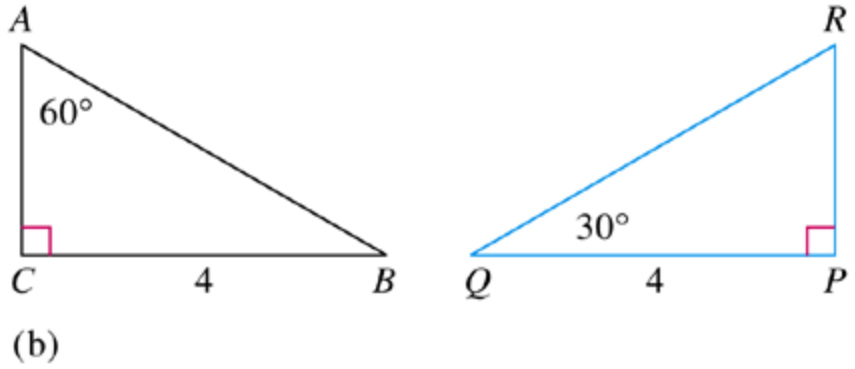
All the above notation can be summarized by saying: Corresponding parts of congruent triangles are congruent, abbreviated CPCTC.

Example 1

Write an appropriate symbolic congruence for each of the pairs.



$\Delta ABC \cong \Delta EFD$



$\Delta ABC \cong \Delta RQP$

Congruence Properties

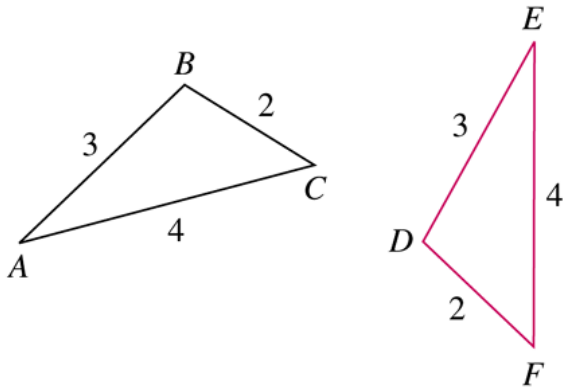
- We know that CPCTC, but is it possible for triangles to be congruent without knowing that all their corresponding parts are congruent?
- Yes! This is where our congruence properties come in. In this section, we will learn about SSS, SAS, and HL. In the next section, we'll learn about ASA and SAA.

Side, Side, Side Congruence Property (SSS)

If the three sides of one triangle are congruent, respectively, to the three sides of a second triangle, then the triangles are congruent.

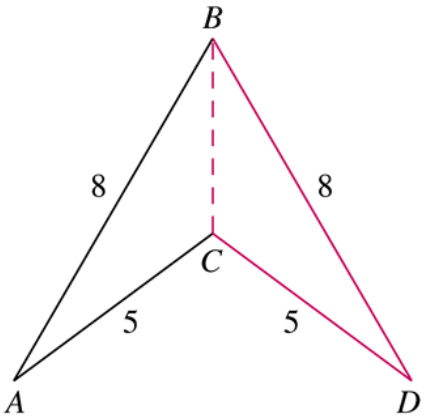
Example 2

Use SSS to explain why the given triangles are congruent.



$\overline{AB} \cong \overline{ED}, \overline{BC} \cong \overline{DF}, \overline{AC} \cong \overline{EF},$

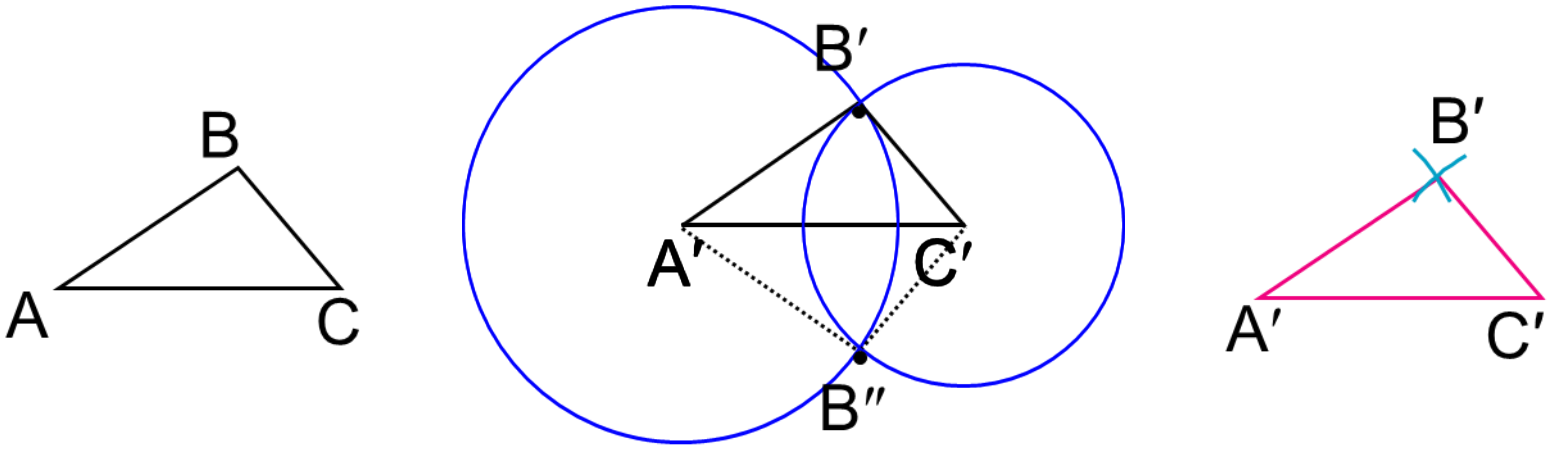
so $\Delta ABC \cong \Delta EDF$ By SSS



$\overline{AB} \cong \overline{DB}, \overline{AC} \cong \overline{DC}, \overline{BC} \cong \overline{BC},$

so $\Delta ABC \cong \Delta DBC$ By SSS

Constructing a Triangle Given Three Sides



First construct $\overline{A'C'}$ congruent to $\overline{AC}$.

Locate the vertex B' by constructing two circles whose radii are congruent to $\overline{AB}$ and $\overline{CB}$.

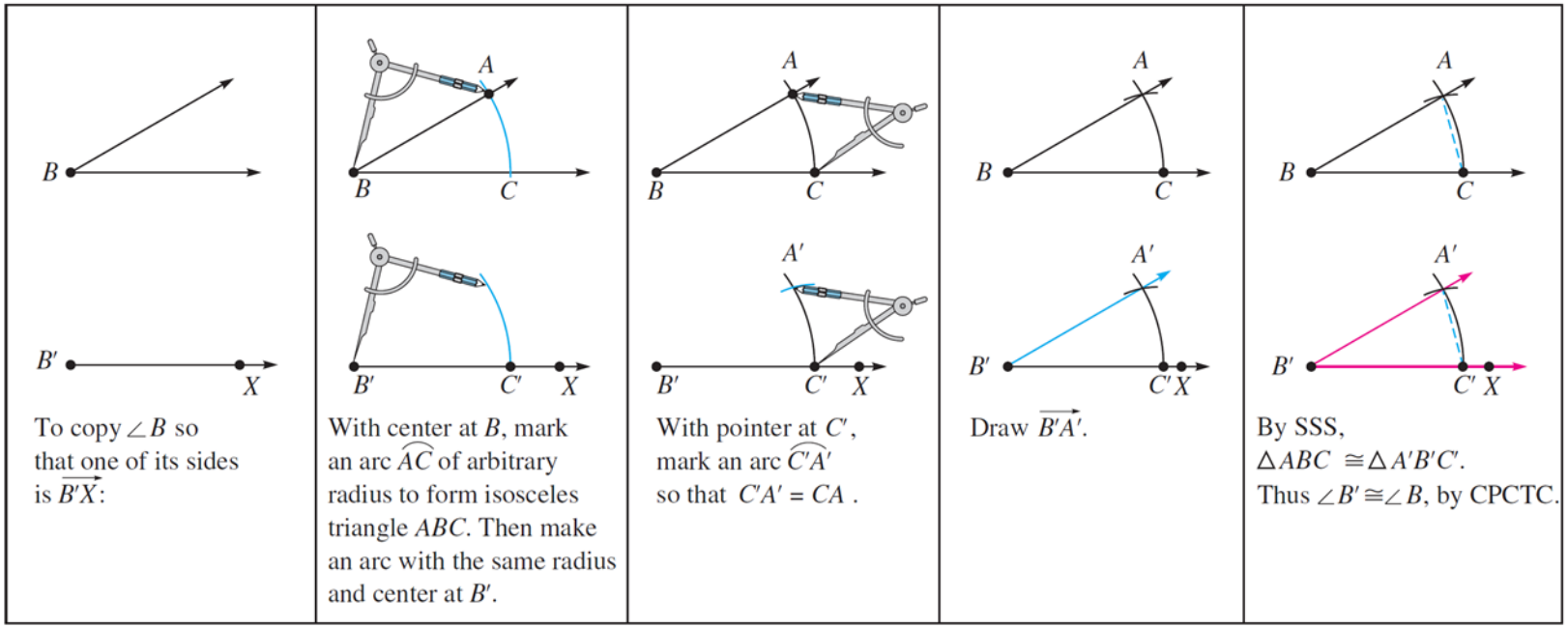
Where the circles intersect (either point) is B'.

Triangle Inequality

The sum of the measures of any two sides of a triangle must be greater than the measure of the third side.

To see this in action, you can find 3 pencils (sticks, straws, pipe cleaners, etc) where 2 of the pencils put end to end are, together, shorter than the 3rd pencil. Then you can use these 3 pencils to try to form a triangle. You will see that it can not be done.

Constructing Congruent Angles



With the compass and straightedge alone, it is impossible in general to construct an angle if given only its measure. Instead, a protractor or a geometry drawing utility or some other measuring tool must be used.

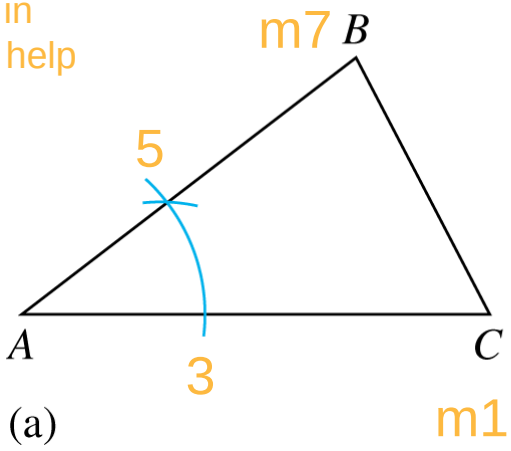
Side, Angle, Side Congruence Property (SAS)

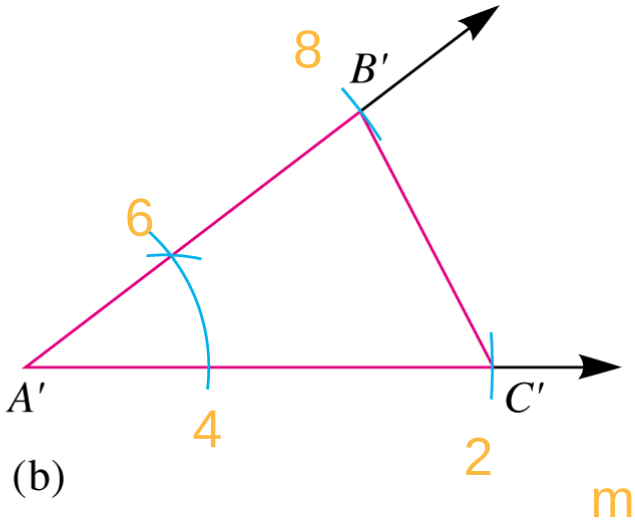
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, respectively, then the two triangles are congruent.

Constructions Involving Two Sides and an Included Angle of a Triangle

First draw a ray with endpoint A', and then construct $\overline{A'C'}$ congruent to $\overline{AC}$. Then construct $\angle A' \cong \angle A$. Mark B' on the side of $\angle A'$ not containing C' so that $\overline{A'B'} \cong \overline{AB}$. Then connect B' and C' to complete the triangle.

The numbers in orange are to help you order the steps.

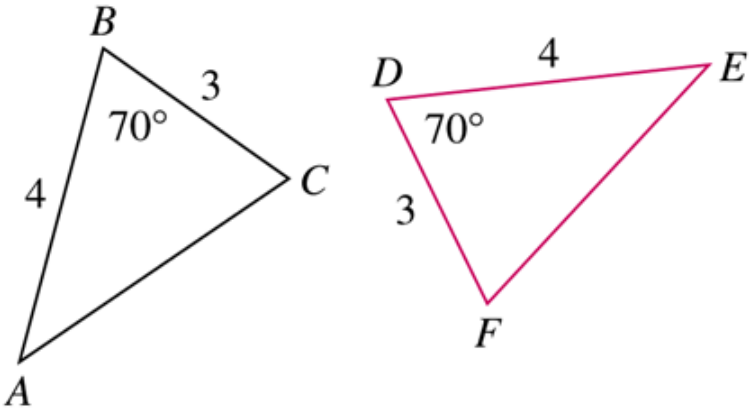




m = measure

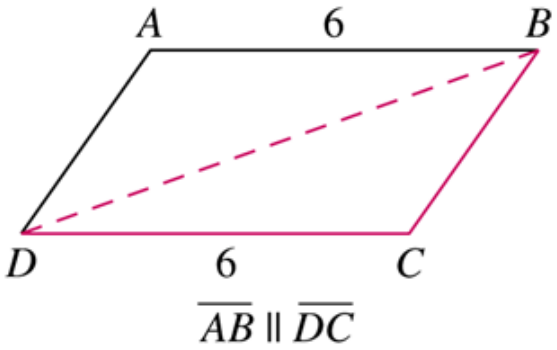
Example 3

Use SAS to show that the given pair of triangles are congruent.



$\overline{AB} \cong \overline{ED}, \angle B \cong \angle D, \overline{BC} \cong \overline{DF}$,

so $\Delta ABC \cong \Delta EDF$



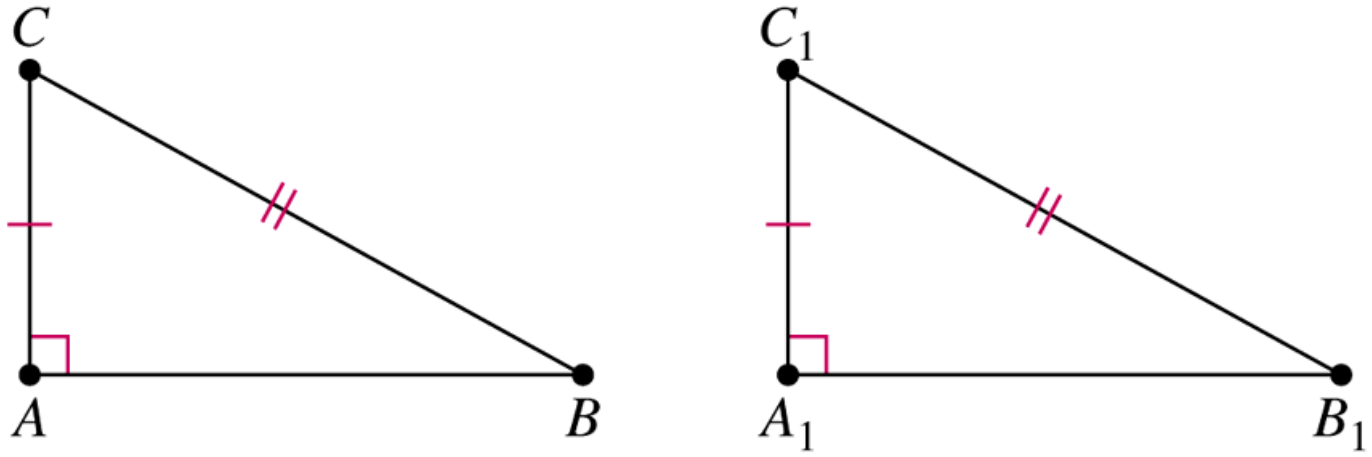
$\overline{AB} \cong \overline{CD}$ and $\overline{DB} \cong \overline{BD}$.

$\angle ABD \cong \angle BDC$ since they are alternate interior angles.

So, $\Delta ABD \cong \Delta CDB$.

Hypotenuse-Leg Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent. (HL is a special case of SSS.)



Selected Triangle Properties (1 of 3)

Any point equidistant from the endpoints of a segment is on the perpendicular bisector of the segment.

Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

Selected Triangle Properties (2 of 3)

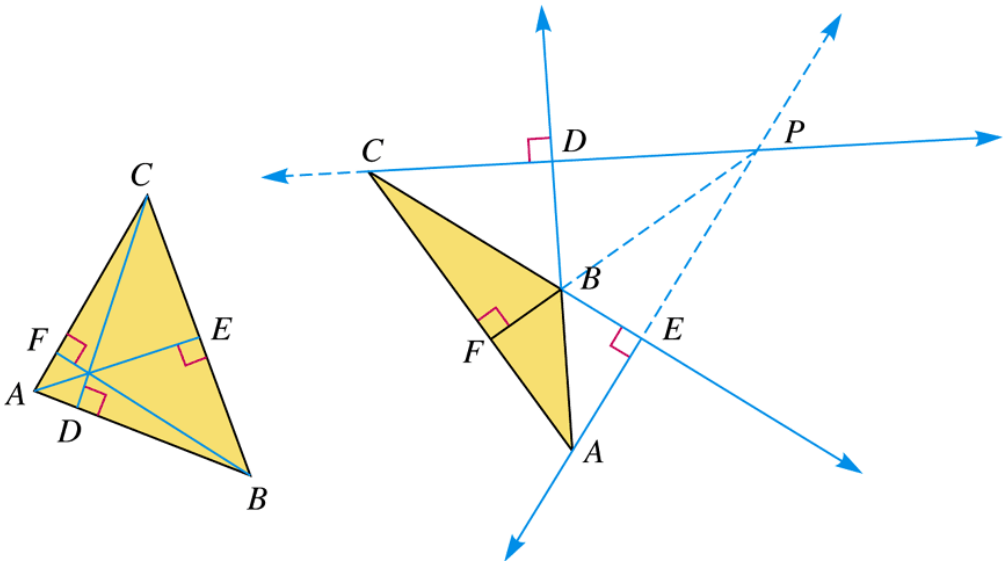
For every isosceles triangle:

The angles opposite the congruent sides are congruent. (Base angles of an isosceles triangle are congruent).

The angle bisector of an angle formed by two congruent sides contains an altitude of the triangle and is the perpendicular bisector of the third side of the triangle.

Selected Triangle Properties (3 of 3)

An **altitude** of a triangle is the perpendicular segment from a vertex of the triangle to the line containing the opposite side of the triangle.

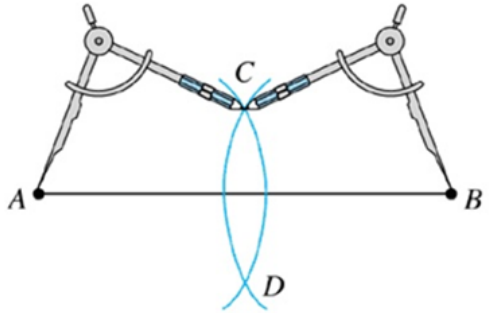


The three altitudes in a triangle intersect in one point.

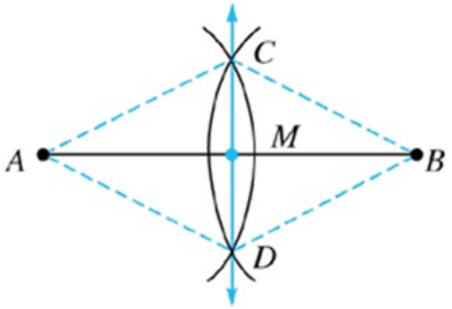
Constructing a Perpendicular Bisector

Bisecting a line segment

To construct a perpendicular bisector of $\overline{AB}$, we make A and B the endpoints of a diagonal of a rhombus.

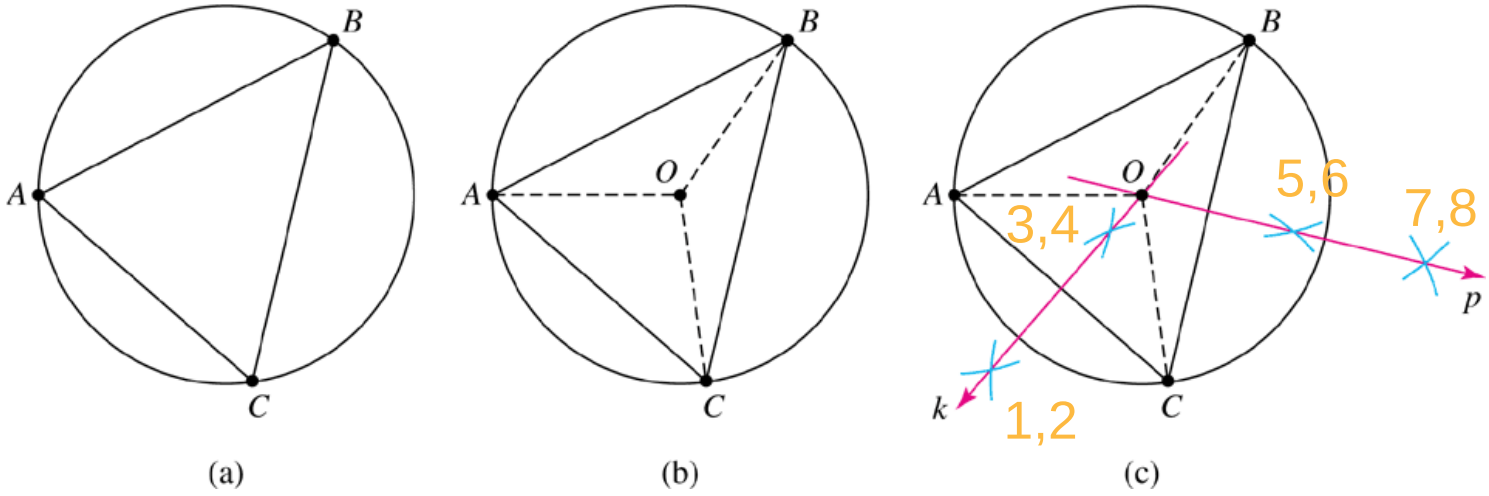


Draw any two intersecting arcs with the same radius (one with center at A, and the other with center at B) to determine $\overline{CD}$, the other diagonal of the rhombus.



The line $\overleftrightarrow{CD}$ connecting the points of intersection of the arcs is the required line. Point M is the midpoint of $\overline{AB}$. $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.

Construction of a Circle Circumscribed About a Triangle



By constructing the perpendicular bisectors of any two sides of triangle ABC, the point where the bisectors intersect is the **circumcenter**.

Links to Videos to Construct…page 1 of 2

- a congruent line segment (1:00) https://www.youtube.com/watch?v=oszaihGRIZ4
- a triangle congruent to a given triangle (aka construct a triangle given three sides (2:04) https://www.youtube.com/watch?v=_c2u2G51ADU
- a congruent angle (1:24) https://www.youtube.com/watch?v=sfO1sAWO0sM

Links to Videos to Construct…(page 2 of 2)

- a perpendicular bisector (1:00) https://www.youtube.com/watch?v=WkA_AstSu7Y
- a circle circumscribed about a given triangle(2:21) https://www.youtube.com/watch?v=qOBkRrE_46k
- a congruent triangle using SAS (3:49) https://www.youtube.com/watch?v=2FPa3-hb-qE&t=7s

This ends section 12-1.